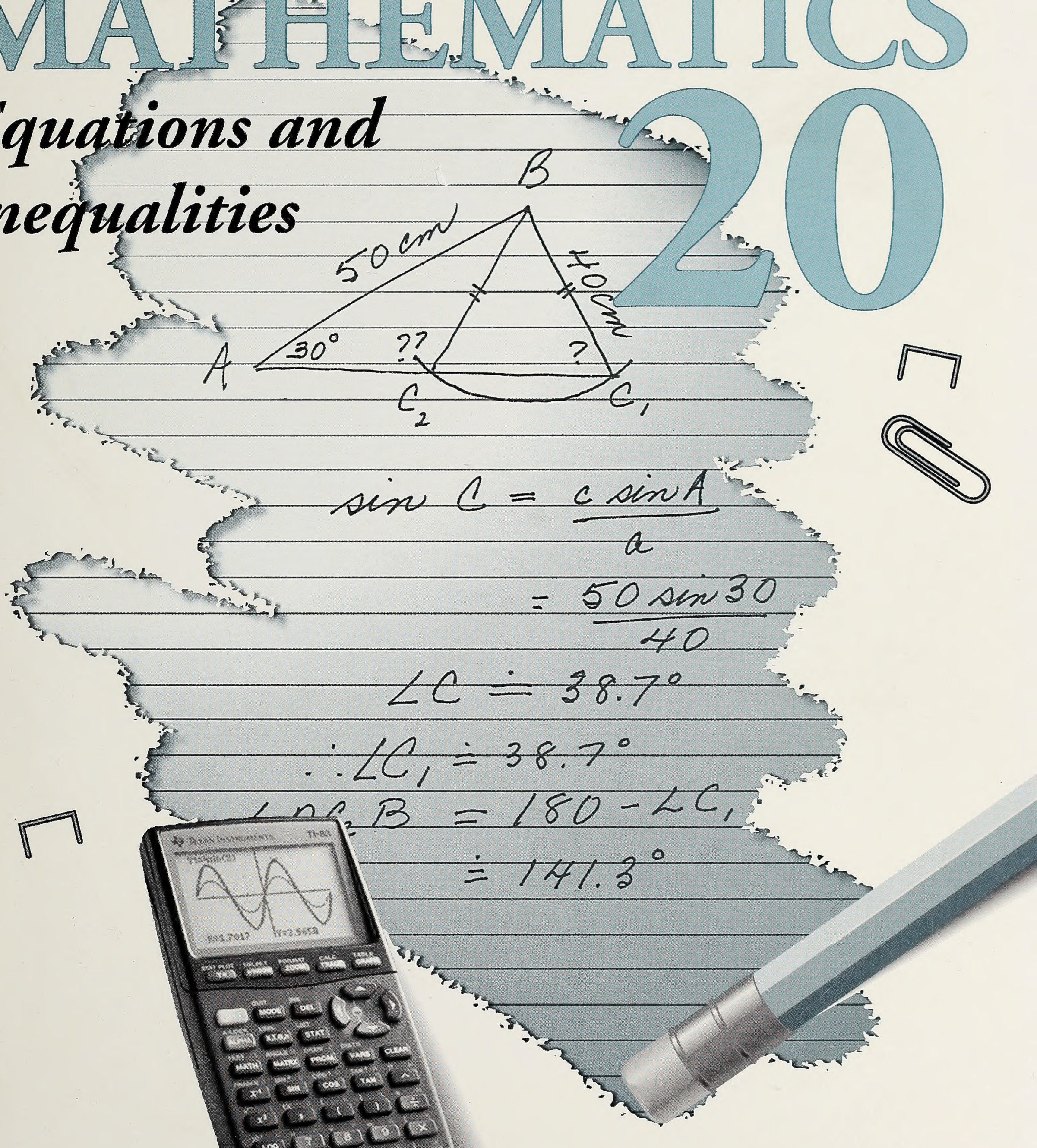
Mathematics: Pure Mathematics 20
Grade: 10, 11, 12
Basic
WCP

Pure MATHEMATICS 20

Module 2

Equations and Inequalities


B
50 cm
40 cm
A
30°
??
?
C_2
C_1
$\sin C = \frac{c \sin A}{a}$
$= \frac{50 \sin 30}{40}$
$\angle C \doteq 38.7°$
$\therefore \angle C_1 \doteq 38.7°$
$\angle AC_2B = 180 - \angle C_1$
$\doteq 141.3°$
TEXAS INSTRUMENTS
TI-83

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Pure MATHEMATICS 20

Module 2

Equations and Inequalities

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Pure Mathematics 20
Student Module Booklet
Module 2
Equations and Inequalities
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

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http://ednet.edc.gov.ab.ca/ltb

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Welcome

Welcome to Module 2. We hope you'll enjoy your study of Equations and Inequalities.

Pure Mathematics 20

Module 1
Consumer Mathematics

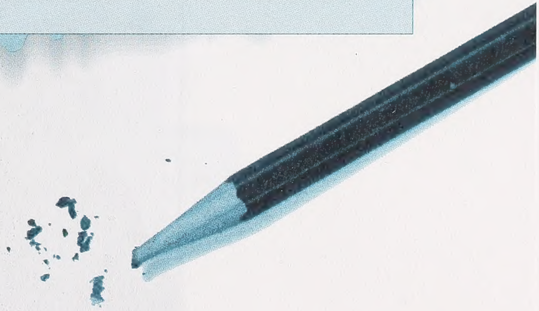
Module 2
Equations and Inequalities

Module 3
Quadratics

Module 4
Functions

Module 5
Mathematical Reasoning and Geometry

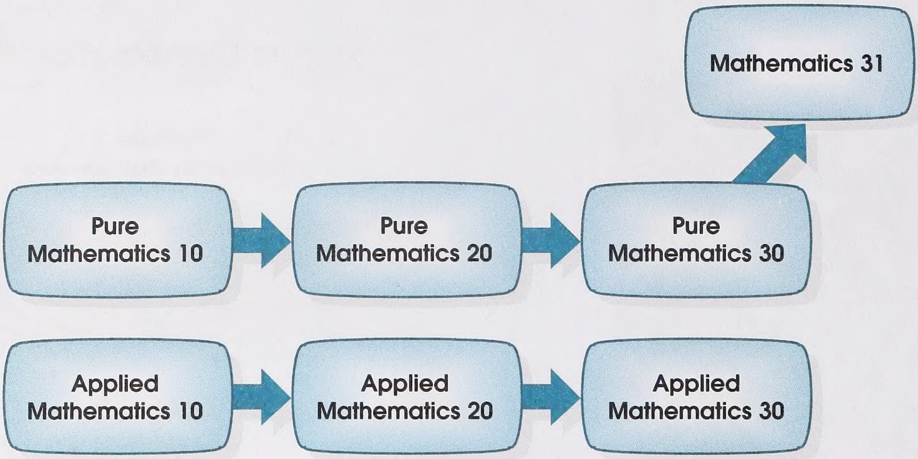
Module 6
Coordinate Geometry and Trigonometry



Pure Mathematics 20 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

High School Mathematics Courses

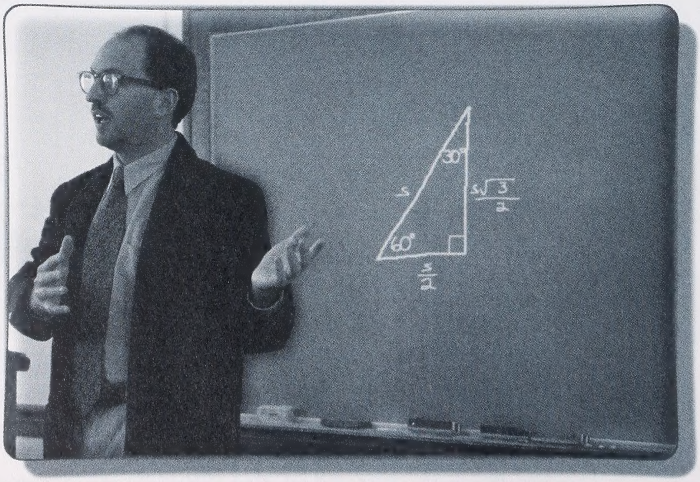
Pure Mathematics 20 is the second course in the Pure Mathematics 10–20–30 sequence of courses. Many students who take the Pure Mathematics 10–20–30 will also choose to take Mathematics 31. Another sequence of courses is Applied Mathematics 10–20–30.



How Do the Sequences Differ?

Each sequence of courses is designed for students with different mathematical strengths and interests. The Pure Mathematics 10–20–30 sequence is intended for students who are strong in mathematical theory and algebra. The Applied Mathematics 10–20–30 sequence is better suited to students who prefer to solve mathematical problems using numerical reasoning or geometry.

Each sequence of courses is designed for students with different career plans. Students who plan to take courses such as commerce, engineering, physics, and engineering technologies in post-secondary education will need to take Pure Mathematics 30 in high school. Some of these students will also need to take Mathematics 31.



Can I Transfer to the Applied Mathematics Sequence of Courses?

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are specific to either applied or pure mathematics courses.

Applied Topics

- linear programming
- data tables and trends
- design and layout
- metric and imperial measure
- data presentation
- vectors and matrices
- periodic, fractal, and recursive patterns
- financial decision making
- costing and design problems

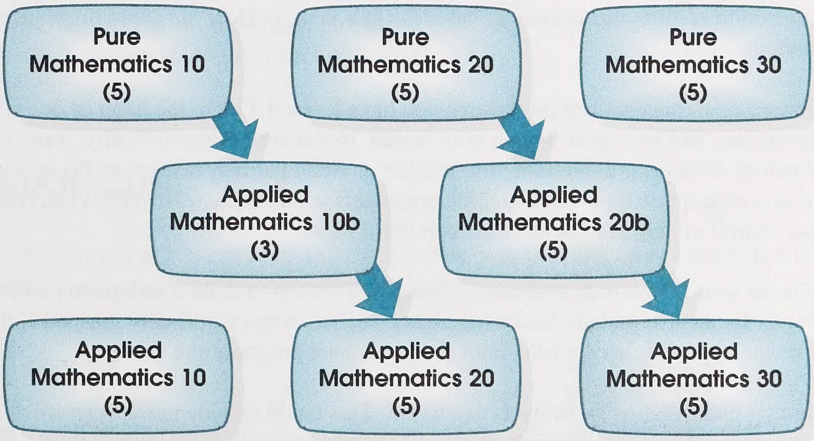
Common Topics

- spreadsheets
- line segments and linear graphs
- scaling
- triangles
- surveys
- financial mathematics
- quadratic functions
- circle geometry
- the bell curve

Pure Topics

- irrational numbers
- exponents
- polynomial and rational expressions
- mathematical expectation
- growth patterns
- linear and non-linear systems
- operations on functions
- mathematical reasoning
- exponential and logarithmic functions
- conics
- combinations
- trigonometric functions

If you want to transfer from the Pure Mathematics 10–20–30 sequence to the Applied Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to the Applied Mathematics sequence of courses after successfully completing Pure Mathematics 10, you may take the 3-credit course called Applied Mathematics 10b or you may take the 5-credit course called Applied Mathematics 10. If you decide to transfer after successfully completing Pure Mathematics 20, you may take the 5-credit course called Applied Mathematics 20b. The two bridging courses are shown in the following diagram:


Pure Mathematics 10 (5)
Pure Mathematics 20 (5)
Pure Mathematics 30 (5)
Applied Mathematics 10b (3)
Applied Mathematics 20b (5)
Applied Mathematics 10 (5)
Applied Mathematics 20 (5)
Applied Mathematics 30 (5)

Mathematical Processes

Throughout the course you will be expected to do the following:

- communicate mathematically
- connect mathematical ideas to everyday experiences and to concepts in other disciplines
- develop and use estimation strategies
- develop and use mental math strategies
- develop and use problem-solving strategies
- reason and justify your thinking
- select and use appropriate technologies as tools to solve problems
- use visualization to assist in processing information, making connections, and solving problems

Mathematical Process

■ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

This icon shows which mathematical process you are expected to perform.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Pure Mathematics 20 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.

You are also encouraged to record your mathematical ideas in a journal. Here are some suggestions for organizing your journal:

- Keep a section on new concepts and procedures you have learned. Get in the habit of describing new concepts, procedures, and strategies in your own words. If you are having difficulty, write your questions in your journal before discussing them with your teacher or study partner. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, or organizational charts) to help you connect mathematical ideas.

- Keep a section on your mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you finally grasped a difficult concept (an “aha!”) or when you first used a calculator or spreadsheet program in a new way.

- Keep a section on mathematics in the everyday world. This could include newspaper articles, cartoons, and descriptions of careers that require a mathematical background.

Resources You Will Need

In order to complete Pure Mathematics 20 successfully, you will need to have access to the following resources, which can be purchased through the Learning Resources Distributing Centre:

- the *MATHPOWER™ 11* textbook, Western Edition, published by McGraw-Hill Ryerson (1999)

- a graphing calculator

  Any of the following graphing calculators may be used: *Texas Instruments* TI-83, TI-83 Plus, TI-82, TI-86, TI-89, TI-92, or TI-92 Plus; *Casio* CFX-9850Ga-Plus, CFX-9850G, CFX-9800G, or FX-9700 series; or *Sharp* EL-9600c, EL-9600, EL-9300, or EL-9200. **Note:** Some of these calculators are no longer commercially manufactured.

- a computer and a spreadsheet program

  Any of the following spreadsheet programs may be used: *ClarisWorks™ 4.0* (or higher) for Macintosh, *ClarisWorks™ 4.0* (or higher) for Windows, *Microsoft® Works 3.0* (or higher) for Windows, or *Microsoft® Access 97* and *Microsoft® Excel 97* for Windows (a part of *Microsoft® Office Professional 95* or *Microsoft® Office Professional 97*).

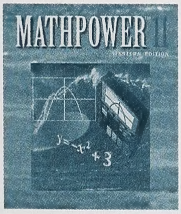
Note: Examples in this course show the TI-83 graphing calculator and the *ClarisWorks™ 4.0* spreadsheet program where applicable. If you have access to a videocassette recorder, you may wish to view the video *The TI-83 Graphing Calculator Video Tutor* to review some of the calculator's features. Check with your distance learning provider concerning the availability of this video.

Finally, you will need the following resources, which can be purchased locally:

- a mathematical instrument set (compass, protractor, ruler, and triangles)
- a binder in which to respond to the questions asked
- a binder for journal writing

Other Visual Cues

In addition to the Mathematical Process icon described earlier, you will find other visual cues throughout this course. Read the explanations given to discover what each icon prompts you to do.



Refer to the textbook.

Use the Internet to explore a topic.

Answer the questions in the Assignment Booklet.

Remember: *Any Internet website address given in this module is subject to change.*

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Solving Linear Inequalities

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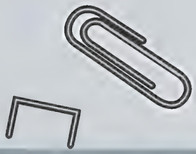

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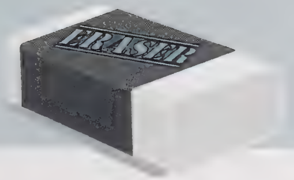


Module Overview

Do you own your own vehicle? If you do, you know that the pleasure, convenience, and independence of vehicle ownership are not without their costs. One of the obvious costs is gasoline—the more you burn, the more you have to pay! Did you know that the cost of gasoline at the pump can be described mathematically by a linear equation? This linear equation relates the volume of gasoline you purchase to the cost shown on the pump.

In Section 1 of this module, you will examine linear equations. You will begin by reviewing methods of solving equations in a single variable and how you can use these equations to model problem situations. You will then build on these skills to explore problems that are more appropriately modelled by systems of linear equations in more than one variable.

In Section 2, you will focus on inequalities derived from linear equations. Again, you will review procedures for solving and graphing linear inequalities in one variable. Then you will explore linear inequalities in two variables—their graphs and their applications.

Module 2: Equations and Inequalities

Section 1:
Solving Linear Equations

Section 2:
Solving Linear Inequalities

Evaluation

The document you are presently reading is called a Student Module Booklet. It will show you, step by step, what to do and how to do it.

This module, Equations and Inequalities, has two sections. Within each section, your work is grouped into activities. Within the activities, there are readings and questions for you to answer. By completing these questions you will construct your own learning, discover mathematical connections, and practise or apply what you have learned. The suggested answers in the Appendix of this Student Module Booklet will provide you with immediate feedback on your progress.

At several points in this module you will be directed to the accompanying Assignment Booklet. In this module you are expected to complete two section assignments and one final module assignment. Your grading in this module is based upon the assignments that you submit for evaluation. The mark distribution is as follows:

Section 1 Assignment	**40 marks**
Section 2 Assignment	**20 marks**
Final Module Assignment	**40 marks**
TOTAL	**100 marks**

Strategies for Completing a Module

Before you begin, organize your materials: Student Module Booklet, textbook, Assignment Booklet, binder, lined paper, graph paper, graphing calculator, pens, pencils, and so on. Make sure you have a quiet area in which to work, away from distractions. Set up your time schedule.

To achieve success in this module, be sure to read all your instructions carefully and work slowly and systematically through the material. Remember, it's the work you do in this Student Module Booklet that will prepare you for your assignments and final test. Try to set realistic goals for yourself each day; and once you've set them, stick to them. Submit your assignments regularly, and don't forget to review your work before handing it in. Careful work habits will greatly increase your chances for success in Pure Mathematics 20.

Good luck!

Section 1

CORBIS/PAUL A. SOUDERS

Lake Louise

Solving Linear Equations

Throughout the world there are special cultural and natural attractions that have significance to all of humanity—not just to the people of the country in which the attraction is found. The United Nations has designated such sites as World Heritage Sites. Within Canada, you can find many World Heritage Sites.

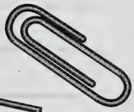
The number of World Heritage Sites in Western Canada and Eastern Canada is 12. The number of World Heritage Sites in Western Canada is twice the number in Eastern Canada. How many World Heritage Sites are in Western Canada? In Eastern Canada?

To solve this problem, you can use the following system of linear equations, where *W* represents the number of sites in Western Canada and *E* represents the number of sites in Eastern Canada.

$W + E = 12$ ①

$W - 2E = 0$ ②

When you find the values of *W* and *E* that fit both equations, you have found the solution to the problem.



In this section, you will solve systems of linear equations using graphs and using algebraic methods—the substitution method and the elimination method. You will then apply systems of equations to solve real-world problems.

Activity 1: Using Algebraic Expressions and Equations

In ancient Egyptian manuscripts, written as early as 1700 B.C., the Egyptians used symbols to represent numbers. These symbols were used to facilitate calculations and make generalizations.

You may not be familiar with the symbols the ancient Egyptians used, but you should be familiar with many of the symbols of modern mathematics. Nevertheless, even with modern notation, an **algebraic expression** modelling a problem situation can appear complicated.

algebraic expression: an expression containing only algebraic symbols and operations—addition, subtraction, multiplication, division, roots, and powers

In this activity, you will begin by reviewing the techniques you studied in Pure Mathematics 10 to simplify algebraic expressions.

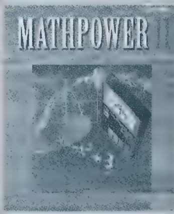
Example

Simplify $-2(3x-5y)+3(2x-3y)$.

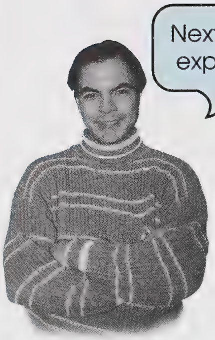
Solution

$$\begin{aligned}-2(3x-5y)+3(2x-3y)&=-6x+10y+6x-9y\\&=y\end{aligned}$$

1. Turn to page 3 of *MATHPOWER™ 11* and answer questions 1, 3, 6, 8, and 10 of "Warm Up."



Compare your responses with those in the Appendix, Section 1: Activity 1.


Next, you will review how algebraic expressions are used in equations.

Recall that an equation is a mathematical statement expressing the equality of two expressions. To solve an equation having only one variable, you must find the number represented by the variable that makes both sides of the equation equal. You can use the guess and check method, but transforming the equation so that the variable is isolated on one side is a more reliable method.

Example

Solve $\frac{x-29}{-5}=\frac{7x+19}{2}$.

Solution

$$\frac{x-29}{-5}=\frac{7x+19}{2}$$

$$-\overset{2}{\cancel{10}}\left(\frac{x-29}{\underset{1}{\cancel{-5}}}\right)=-\overset{-5}{\cancel{10}}\left(\frac{7x+19}{\underset{1}{\cancel{2}}}\right)$$ ← Multiply both sides by -10, the LCD.

$$2x-58=-35x-95$$ ← Simplify.

$$37x=-37$$

$$x=-1$$

2. Turn to page 3 of *MATHPOWER™ 11* and answer questions 19, 21, 23, and 24 of "Warm Up."

Compare your responses with those in the Appendix, Section 1: Activity 1.

In the preceding exercise, you were able to find the value of the variable that makes the left side of the equation equal the right side of the equation. However, when an equation has more than one variable, you cannot find a unique numerical value for either variable. Solving such an equation for one variable simply means expressing one of the variables in terms of the others. The resulting expression can be used to find many sets of values that satisfy the equation.

Example

Solve $3w - 2z = 30$ for w.

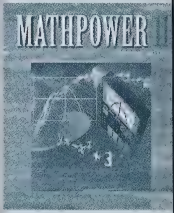
Solution

$$3w - 2z = 30$$

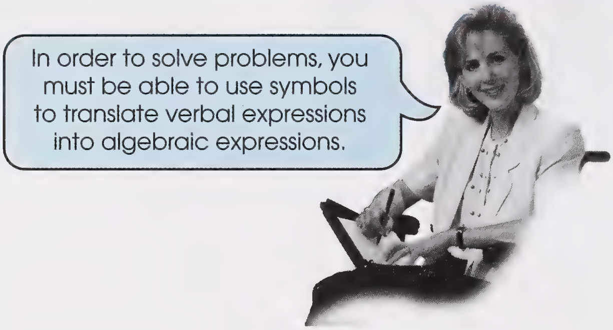
$$3w = 2z + 30 \quad \longleftarrow \text{Add } 2z \text{ to both sides.}$$

$$w = \frac{2z + 30}{3} \quad \longleftarrow \text{Divide both sides by 3.}$$

3. Turn to page 3 of *MATHPOWER™ 11* and answer questions 25, 27, 29, and 31 of "Warm Up."



Compare your responses with those in the Appendix, Section 1: Activity 1.


In order to solve problems, you must be able to use symbols to translate verbal expressions into algebraic expressions.

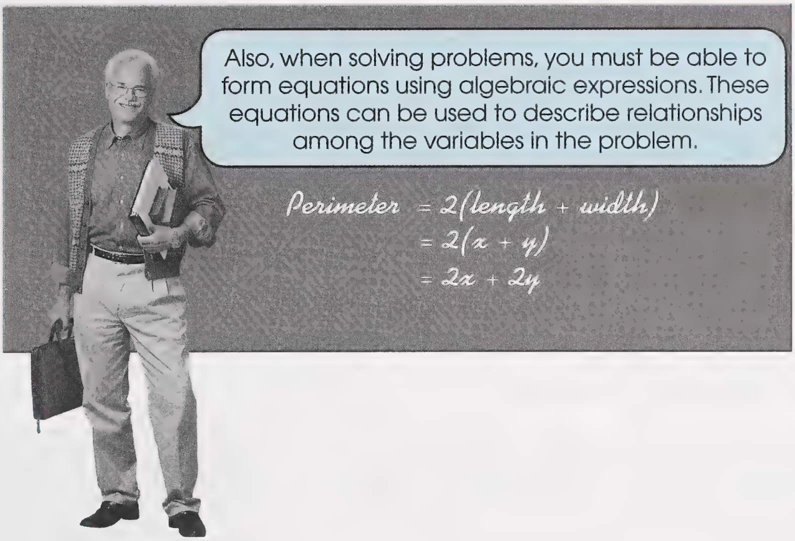
Example

The length (in metres) of a tennis court is x, and its width (in metres) is y. Write an algebraic expression that represents the perimeter.

Solution

$$\begin{aligned} \text{Perimeter} &= 2(\text{length} + \text{width}) \\ &= 2(x + y) \\ &= 2x + 2y \end{aligned}$$

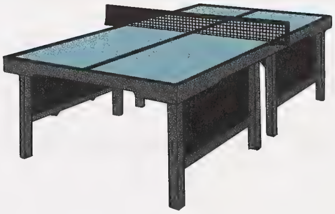
The perimeter of the tennis court is $2x + 2y$.



Also, when solving problems, you must be able to form equations using algebraic expressions. These equations can be used to describe relationships among the variables in the problem.

$$\begin{aligned}\text{Perimeter} &= 2(\text{length} + \text{width}) \\ &= 2(x + y) \\ &= 2x + 2y\end{aligned}$$

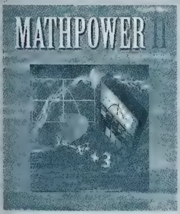
Example



The length (in centimetres) of a regulation table for table tennis is x, and the width (in centimetres) is y. The perimeter of the table is 853 cm. Write an equation to show how the variables x and y are related.

Solution

$$\begin{aligned}\text{Perimeter} &= 2x + 2y \\ 853 &= 2x + 2y\end{aligned}$$



4. Turn to “INVESTIGATING MATH” on page 16 of *MATHPOWER™ 11* and answer the following:

   a. questions 1, 4, and 6 of Investigation 1, “Expressions in Two Variables”
   b. questions 5, 6, and 8 of Investigation 2, “Equations in Two Variables”

Compare your responses with those in the Appendix, Section 1: Activity 1.

So far, you have been working with single equations. However, two or more equations may arise from the same problem situation. A **system of equations** can be a powerful tool to solve these problems.

system of equations: a group of equations that are to be considered at the same time

Example

Genine has 14 coins of only dimes and nickels. The value of the coins is \$1.35. Write a system of two equations in x and y that describes the relation involving the numbers of coins and the relation involving the value of the coins.

Solution

Let x be the number of nickels, and let y be the number of dimes.

There are $x + y$ coins altogether.

$\therefore\ x + y = 14$

The value (in cents) of the nickels is $5x$, and the value (in cents) of the dimes is $10y$.

$\therefore\ 5x + 10y = 135$

\$1.35 = 135¢

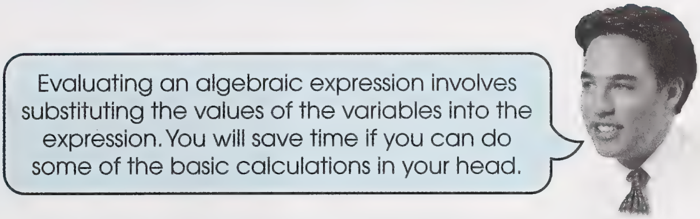
The system of equations is as follows:

$x + y = 14$ ①
$5x + 10y = 135$ ②

5. Turn to page 17 of *MATHPOWER™ 11* and answer questions 2, 4, 7, and 8 of Investigation 3, "Systems of Equations."

Compare your responses with those in the Appendix, Section 1: Activity 1.

Now Try This

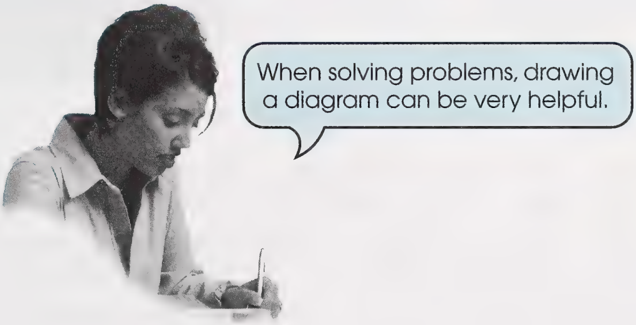

Evaluating an algebraic expression involves substituting the values of the variables into the expression. You will save time if you can do some of the basic calculations in your head.

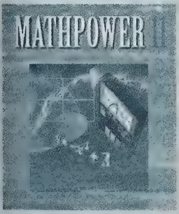
Mathematical Process

☐ Communication
☐ Connection
☐ Estimation
☒ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

6. Turn to page 3 of *MATHPOWER*™ *11* and answer questions 2, 4, 6, 8, and 10 of "Mental Math: Evaluating Expressions."

Compare your responses with those in the Appendix, Section 1: Activity 1.


When solving problems, drawing a diagram can be very helpful.



7. Turn to page 15 of *MATHPOWER*™ *11* and answer questions 6, 8, and 12 of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 1: Activity 1.

Looking Back

Mathematical Process

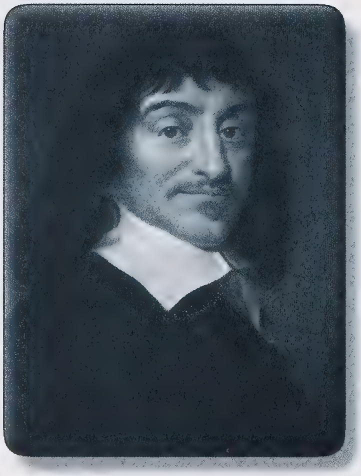
☒ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

In this activity, you simplified algebraic expressions, represented problem situations with algebraic expressions and equations, and then solved these problem situations.

In your journal, explain how solving an equation with one variable compares to solving an equation with two variables. Also, in your own words, describe a quantity that can be represented by an algebraic expression with two variables; then, write the algebraic expression for the quantity you described.

Activity 2: Solving Systems of Equations by Graphing

René Descartes, a French philosopher of the seventeenth century, introduced the idea of using a coordinate plane to visualize equations.


CORBIS-BETTMANN

René Descartes

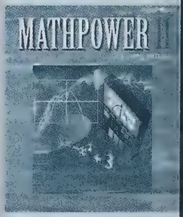
When you graph an equation, you create a picture that allows you to see the properties of that equation. In the coordinate plane, an equation appears as a line or curve. Each point on the line or curve corresponds to an ordered pair that satisfies the equation.

In this activity, you will explore how the graph of a system of equations can be used to determine the solution of that system.

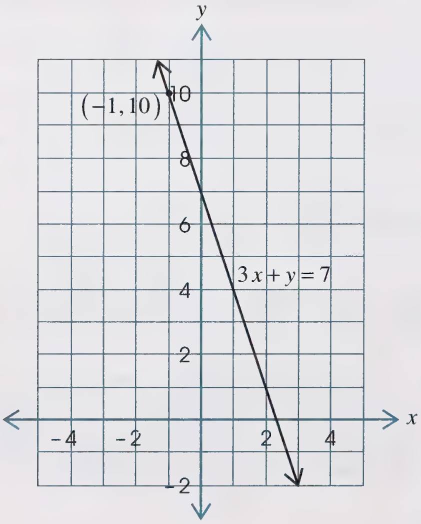
Ordered Pairs and Solutions to Equations

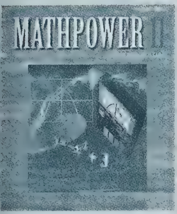
Recall that if you substitute the *x*- and *y*-values of an ordered pair into an equation with two variables and it makes the left side of the equation equal the right side, then the ordered pair is said to satisfy the equation. Applying this idea, the ordered pair $(-1, 10)$ satisfies the equation $3x + y = 7$ if the left side of the equation equals the right side when you replace x with -1 and y with 10.

Turn to "INVESTIGATING MATH" on page 4 of *MATHPOWER*™ *11* and read Investigation 1, "Ordered Pairs and One Equation."



Note that the point $(-1, 10)$ is on the graph of $3x + y = 7$.

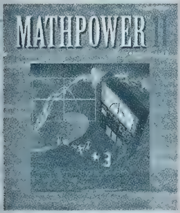




1. Answer questions 1.c., 1.d., 1.e., 2.b., and 2.c. of Investigation 1, "Ordered Pairs and One Equation" on page 4 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 2.

You've just reviewed how to determine if an ordered pair satisfies an equation. Suppose you had a system of two equations in two unknowns. How many ordered pairs do you think would satisfy both equations? What would you call these ordered pairs?

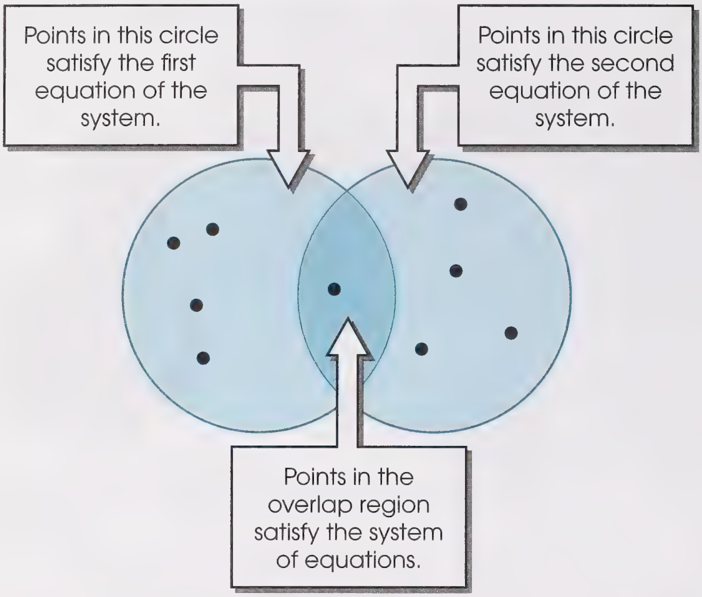


Turn to "INVESTIGATING MATH" on page 4 of *MATHPOWER*™ *11* and read Investigation 2, "Ordered Pairs and Two Equations."

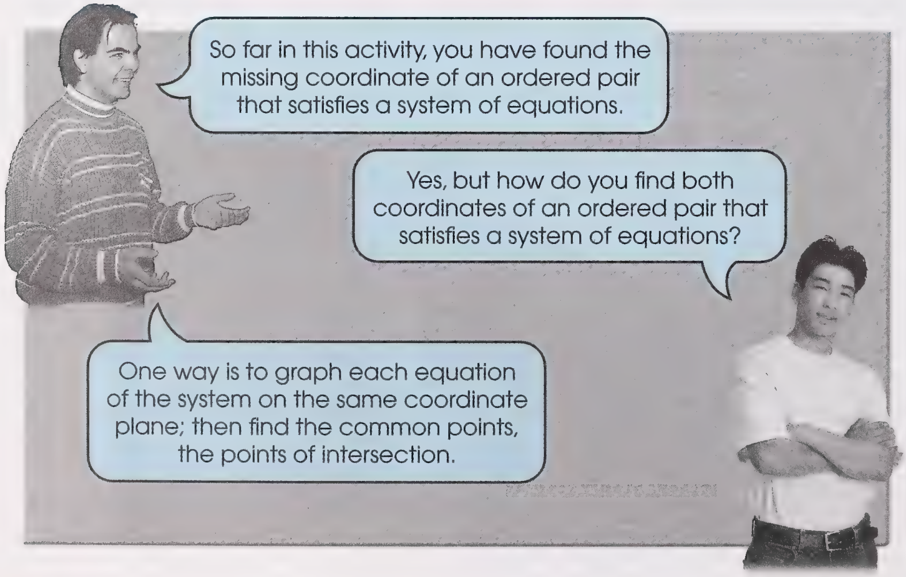
2. Answer the following questions on page 5 of the textbook:
   - **a.** questions 1.b., 1.d., 2.b., and 2.d. of Investigation 2, "Ordered Pairs and Two Equations"
   - **b.** questions 2 to 4 of Investigation 3, "Problem Solving"

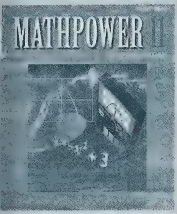
Compare your responses with those in the Appendix, Section 1: Activity 2.

The following Venn diagram shows how the points that satisfy individual equations of a system of equations compare to the points that satisfy the system of equations. For some systems, the overlap region may not have any points; for others, there may be many.


Points in this circle satisfy the first equation of the system.
Points in this circle satisfy the second equation of the system.
Points in the overlap region satisfy the system of equations.

Graphs and Solutions to Systems of Equations


So far in this activity, you have found the missing coordinate of an ordered pair that satisfies a system of equations.
Yes, but how do you find both coordinates of an ordered pair that satisfies a system of equations?
One way is to graph each equation of the system on the same coordinate plane; then find the common points, the points of intersection.

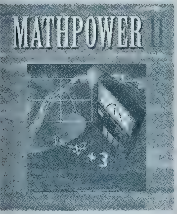


3. Turn to page 6 of *MATHPOWER™ 11* and answer the following:

- **a.** questions a., b., and c. of "Explore: Use a Graph"
- **b.** questions 1, 2, 3, 5.a., and 5.b. of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 2.

When you found the ordered pairs that satisfy a pair of equations, you solved a linear system. In the next part of this activity, you will refine your techniques for solving linear systems.

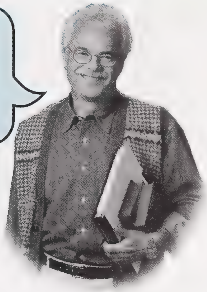


Turn to page 6 of *MATHPOWER™ 11* and read from the red line to the red line on page 9, working through Examples 1 to 3.

4. Answer the following questions on pages 11 and 12 of the textbook:

- **a.** questions 2, 4, 7, 8, 14, 16, 22, and 24 of "Practice"
- **b.** questions 32, 34, 38, 40, and 42.b. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.



In the previous set of questions, each pair of lines intersected at exactly one point. What would you do if the linear system represented a pair of parallel lines? or the same line?

Turn to page 9 of *MATHPOWER™ 11* and read from the red line to the bottom of page 10, working through Examples 4 and 5.

5. Answer the following questions on pages 11 and 12 of the textbook:

- **a.** questions 26, 28, and 29 of "Practice"
- **b.** questions 44 and 46 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Now Try This

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

You may have heard the expression, “A picture is worth a thousand words.” This saying certainly applies to the use of diagrams in problem solving. Diagrams can help you understand a problem and solve it.

Turn to “PROBLEM SOLVING” on page 14 of *MATHPOWER™ 11* and read “Use a Diagram.”

If you need convincing that the corkscrew path of the vine can be compared to the hypotenuse of a right triangle, do this:

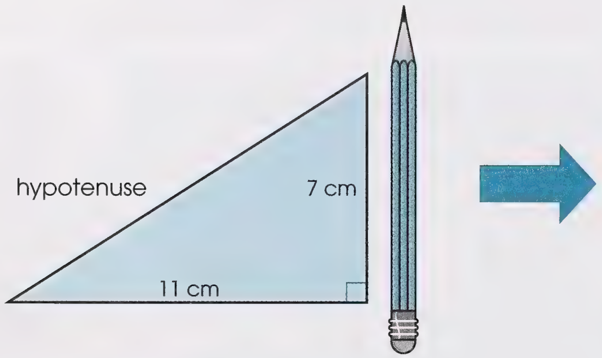
- Cut a corner off a sheet of loose-leaf paper, making the cut as straight as possible.

  Start your cut about 11 cm from a corner and continue so that the cut meets the other edge, about 7 cm from the same corner. This gives you a right triangle.

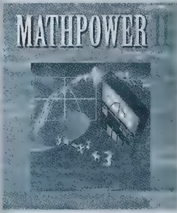
- Starting at the right-angle end of the triangle, wind the paper around a pencil, keeping the shorter side oriented in the direction of the pencil length.

You’ll see that the hypotenuse of the triangle forms a **cylindrical helix** or corkscrew path.

cylindrical helix: a curve ascending a cylinder at a constant angle



6. Turn to page 15 of *MATHPOWER™ 11* and answer questions 1, 2, 3, 5, and 11 of “Applications and Problem Solving.”



Compare your responses with those in the Appendix, Section 1: Activity 2.

Looking Back

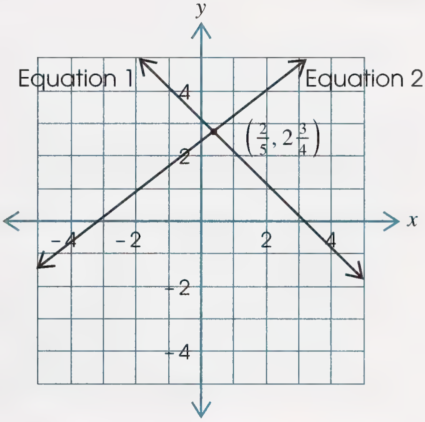
In this activity, you reviewed what is meant by the solution of an equation and extended the definition to the solution of a system of equations. You then found solutions to systems of equations by graphing. Following this, you applied diagrams to problem solving.

In your journal, use words and a diagram to describe systems of equations that have just one ordered pair in their solutions and systems of equations that have no ordered pairs in their solutions. Next, write a problem that can be solved using a diagram. Have another student taking Pure Mathematics 20 solve your problem.

Mathematical Process

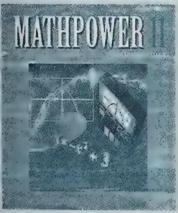
- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Activity 3: Solving Systems of Equations by Substitution



Graphing allows you to visualize the equations of a system and the solution to the system. However, graphing cannot be used in every case to determine the solution of a system. For example, if the ordered pair $\left(\frac{2}{5}, 2\frac{3}{4}\right)$ satisfies the equations of a system (like the one shown in the graph), the x- and y-values cannot be determined easily from the point of intersection of the lines.

Algebraic methods of solving systems of equations will give you exact values, and algebraic methods are often more convenient. Solving systems by substitution is one such algebraic method.



Turn to page 21 of *MATHPOWER*™ *11* and read "Solving Systems of Linear Equations by Substitution" and "Explore: Use the Equations."

1. Refer to the system of equations in “Explore: Use the Equation” to answer the following:

   a. What does equation 1 tell you about the relation of numbers of tigers and leopards?

   b. What relation do you see from equation 2?

Compare your responses with those in the Appendix, Section 1: Activity 3.

From equation 1, you can see that $t = 15\ell$. Substitute this value for t into equation 2.

$$15\ell + \ell = 480$$

This equation has just one variable now; thus, it can be solved for ℓ.

$$\begin{aligned} 16\ell &= 480 \\ \ell &= 30 \end{aligned}$$

Either equation of the system can be rewritten with the value of 30 for ℓ.

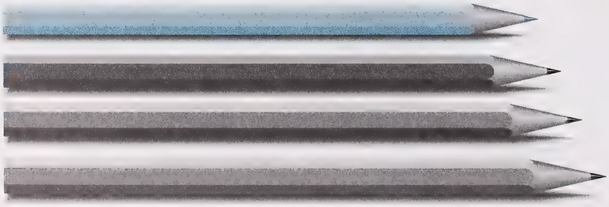
Now, substitute $\ell = 30$ into equation 2 and solve for t.

$$\begin{aligned} t + 30 &= 480 \\ t &= 450 \end{aligned}$$

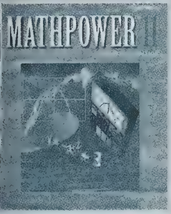
There are 450 Siberian tigers and 30 Amur leopards.

2. Turn to page 21 of *MATHPOWER™ 11* and answer question 4 of “Inquire.”

Compare your response with the one in the Appendix, Section 1: Activity 3.



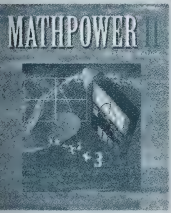
In the preceding question, you obtained the ordered pair by using equation 1 to solve for one variable in terms of the other. Would it matter if equation 2 were used?



3. Turn to page 21 of *MATHPOWER™ 11* and answer questions 5 and 7.c. of "Inquire."

Compare your responses with those in the Appendix, Section 1: Activity 3.

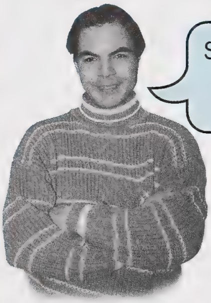
The method of solving a linear system by substitution gives you an equation in one variable, which can be solved easily using techniques you studied in previous mathematics courses.

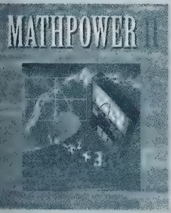


Turn to page 22 of *MATHPOWER™ 11* and read from the top of the page to the red line, working through Example 1.

4. Answer questions 10, 12, 13, and 15 of "Practice" on page 26 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 3.


So far, your solutions involved only integers; solutions don't always come out so neatly.



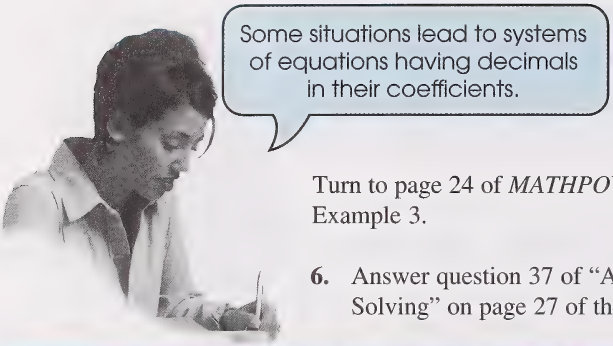
Turn to page 22 of *MATHPOWER™ 11* and read from the red line to the bottom of page 23, working through Example 2.

As you can see from Example 2, graphing the system of equations may not indicate the exact solution.

5. Answer the following questions on page 26 of the textbook:

   a. questions 14, 17, 21, 22, and 24 of "Practice"
   b. question 29.c. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 3.

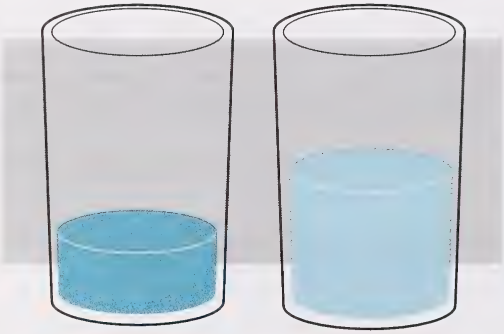

Some situations lead to systems of equations having decimals in their coefficients.

Turn to page 24 of *MATHPOWER*™ *11* and work through Example 3.

6. Answer question 37 of “Applications and Problem Solving” on page 27 of the textbook.

Compare your response with the one in the Appendix, Section 1: Activity 3.

Suppose you have a solution and add an equal amount of water to it. The concentration of the diluted solution will be half the concentration of the original solution. Generally speaking, when mixing equal amounts of solutions with different concentrations, the resulting solution has a concentration equal to the average of the initial concentrations.



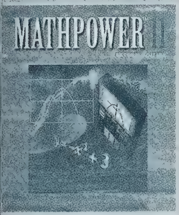
If you have two solutions with different concentrations and you want to mix portions of the two solutions to prepare a certain amount of solution at a target concentration, how much of each would you use? A system of equations can help answer this question.

Turn to page 25 of *MATHPOWER*™ *11* and work through Example 4.

7. Answer questions 40 and 41 of “Applications and Problem Solving” on page 27 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 3.

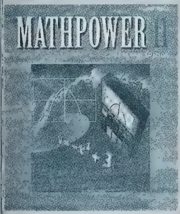
Systems of equations can be used in a variety of situations, not only for investing money and mixing solutions.



8. Turn to pages 26 and 27 of *MATHPOWER™ 11* and answer questions 30, 32, 33, 35, 36, 42, and 46.

Compare your responses with those in the Appendix, Section 1: Activity 3.

Now Try This



Turn to "CAREER CONNECTION" on page 30 of *MATHPOWER™ 11* and read "Chemistry."

Mathematical Process
☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

You know from experience that sugar dissolves in water. You can easily stir one teaspoon of sugar into a cup of water. However, if you keep adding sugar, you will reach a point at which it will no longer dissolve in water and just settle on the bottom of the glass. The amount of sugar you can add to the water before it begins collecting at the bottom is related to the solubility of sugar in water. Different substances have different solubilities in water.

9. Turn to "CAREER CONNECTION" on page 30 of *MATHPOWER™ 11* and answer questions 1 and 4 of Investigation 1, "Comparing Solubilities."

Compare your responses with those in the Appendix, Section 1: Activity 3.

Looking Back

Mathematical Process
☒ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

In this activity, you found solutions to systems of equations using the algebraic method of substitution. You then applied this method to a variety of real-world situations. Finally, you gained an appreciation for careers in chemistry and the role of mathematics in the chemistry of solutions.

You now have solved systems of equations by substitution and by graphing. Which method do you prefer? Write your thoughts in your journal.

Activity 4: Solving Systems of Equations by Elimination

Besides substitution, there is another algebraic method of solving systems of equations—the elimination method. This method is based on the principle that the sum or difference of two equations produces another equation that is satisfied by the same ordered pair as the original system.

For example, suppose you wanted to determine the speed (in km/h) the boat in the photograph can travel in still water. You are told that travelling upstream, against the current, it can cruise at 60 km/h. You are also told that travelling downstream, with the same current, it can cruise at 65 km/h.

If b represents the speed of the boat and c represents the speed of the current, this situation can be modelled by the following pair of equations:

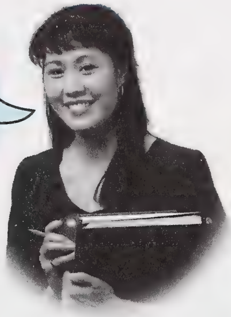
$b - c = 60$ ①

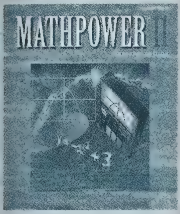
$b + c = 65$ ②

If you add the two equations, the result is a third equation that allows you to find the boat's speed immediately.

$$\begin{aligned} b - c &= 60 \\ \underline{b + c} &\underline{= 65} \\ 2b + 0c &= 125 \\ 2b &= 125 \\ b &= 62.5 \end{aligned}$$

The boat is travelling 62.5 km/h in still water.


In this activity, you will use this principle to solve linear systems.

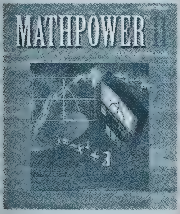


Turn to page 34 of *MATHPOWER™ 11* and read "Solving Systems of Linear Equations by Elimination."

1. Answer the following questions on page 34 of the textbook:

- **a.** questions a. to d. of "Explore: Use the Equations"
- **b.** questions 1 to 6 of "Inquire"

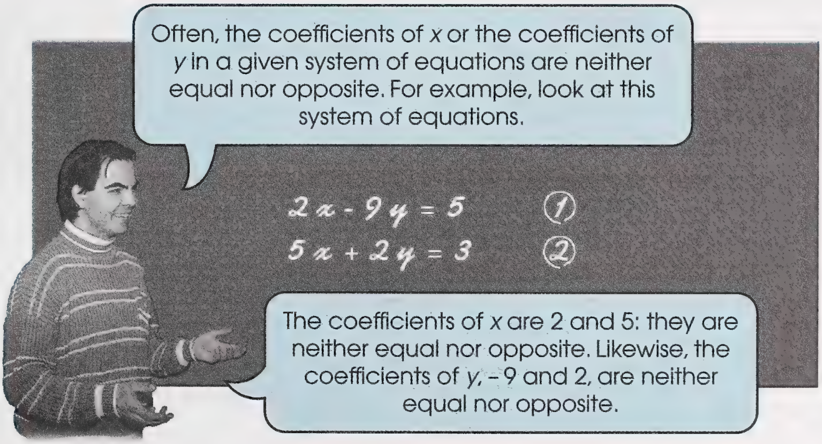
Compare your responses with those in the Appendix, Section 1: Activity 4.



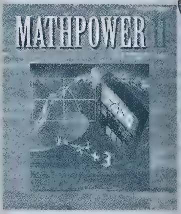
Now, turn to page 34 of *MATHPOWER™ 11* and read from the red line to the bottom of page 35, working through Examples 1 and 2.

2. Answer questions 1, 2, 4, and 6 of "Practice" on page 38 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 4.


Often, the coefficients of x or the coefficients of y in a given system of equations are neither equal nor opposite. For example, look at this system of equations.
$2x - 9y = 5$ ①
$5x + 2y = 3$ ②
The coefficients of x are 2 and 5: they are neither equal nor opposite. Likewise, the coefficients of y, −9 and 2, are neither equal nor opposite.

In such cases, multiplying one or both of the equations can give you an equivalent system that can then be solved simply by subtraction or addition.



Turn to page 36 of *MATHPOWER™ 11* and work through Example 3.

3. Answer the following questions on pages 38 to 40 of the textbook:

a. questions 9 to 13, 18, 22, 24, 26, and 29 of "Practice"

You may want to review your journal entry for the previous activity.
There you discussed solving systems by graphing and by substitution.
You now have a third method in your repertoire.

b. questions 43, 45, and 50 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 4.

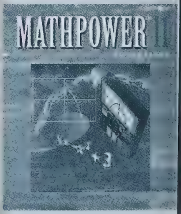
In some systems of equations, the variables appear within numerators of fractions or have rational coefficients. For example, look at the following system of equations:

$$\frac{x+1}{2} - \frac{3y}{5} = 4 \quad ①$$

$$\frac{2x}{3} - y = \frac{5}{12} \quad ②$$

You can convert such a system into an equivalent system without fractions; then you can solve the converted system using the elimination method, which you have already applied.

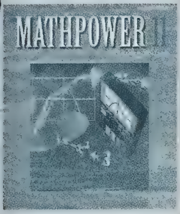
Turn to pages 36 and 37 of *MATHPOWER™ 11* and work through Example 4.



4. Answer the following questions on page 39 of the textbook:

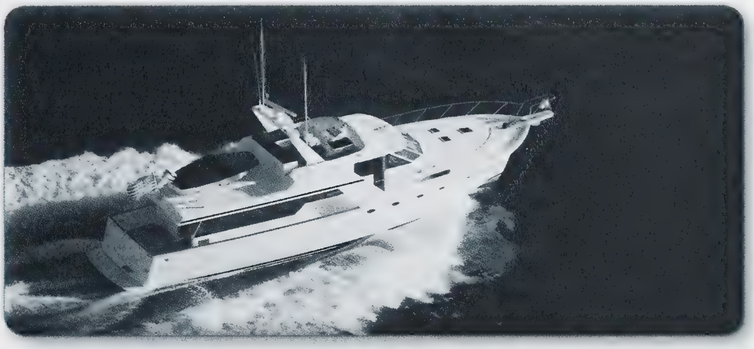
a. questions 31 and 35 of "Practice"
b. questions 44 and 46 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 4.



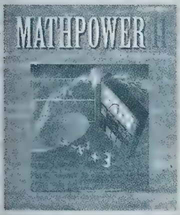
Turn to page 37 of *MATHPOWER™ 11* and work through Example 5. In this example, a chart is used to organize the information. In similar applications, you will find the chart to be an effective way of sorting out the details.

So, whenever you are solving a problem in which a vessel or an aircraft moves relative to currents or winds, use a chart to simplify the problem!



5. Answer questions 47 and 48 of “Applications and Problem Solving” on page 39 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 4.



Turn to page 38 of *MATHPOWER™ 11* and work through Example 6. As in the previous example, speed is involved. This time, however, the speeds are only with respect to the ground and not with respect to a moving medium. Also, notice that a system of rational equations is used to determine the answer to the question.

The example you just worked through involves speed. Speed is a special rate—that of distance versus time. Keep in mind that the approach used in Example 6 can be adapted to problems involving rates other than speed, such as fuel consumption, population growth, and interest on investment.

6. Answer questions 49, 51, 52, 56, 57, and 58.a. of “Applications and Problem Solving” on pages 39 and 40 of the textbook.

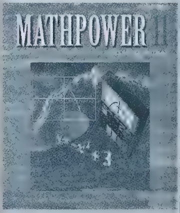
Compare your responses with those in the Appendix, Section 1: Activity 4.

Now Try This

Mathematical Process

☐ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☒ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

Imagine how many cars could fill their gas tanks from the aviation fuel used by just one jumbo jet. Although jet fuel has a higher octane rating than regular gasoline and has other characteristics making it unsuitable for motor vehicles, if you based the answer solely on the volumes of fuel involved, 2500 cars would be a reasonable estimate. Questions such as these, ones involving quantities, are called Fermi problems.



Turn to "PROBLEM SOLVING" on page 48 of *MATHPOWER™ 11* and read "Solve Fermi Problems."

7. Answer question 2 of "Applications and Problem Solving" on page 49 of the textbook.

8. If you allowed $1\ m^2$ standing room per person, what percentage of Alberta's surface area would be needed to provide standing room to all of Earth's population?

Compare your responses with those in the Appendix, Section 1: Activity 4.

Looking Back

Mathematical Process

☒ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

In this activity, you solved systems of equations using the elimination method. You then examined Fermi problems and the methods used to solve them.

In your journal, write an example of a system of equations that can be solved by graphing, by substitution, and by elimination. Then give a complete written solution for each method. When is substitution more appropriate than elimination?

Activity 5: Solving Systems of Equations in Three Variables

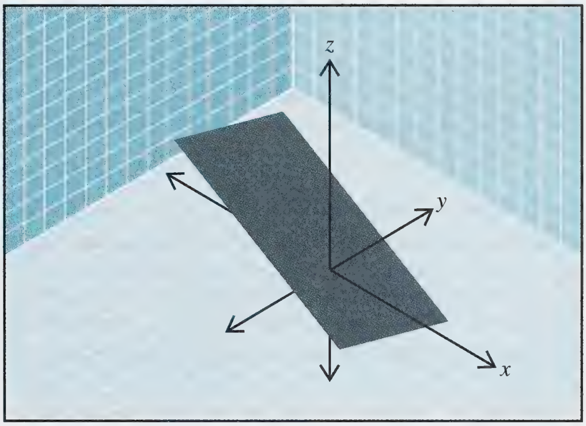
Mathematical Process

■ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
☒ Visualization

The systems of linear equations you have solved so far have all involved two equations in two unknowns. You will now solve systems of three equations in three unknowns.

You know that linear equations having two variables can be graphed in the coordinate plane as a line. What about an equation with three varibles?

A linear equation having three variables can also be represented graphically; it can be represented as a flat surface in three-dimensional space. The following illustration shows what the graph of $z = -x - y$ looks like.



The surface of the graph extends infinitely in all directions. Here, only a limited portion near the origin is shown so the axes and the background remain visible. From this illustration, the surface tilts upward and to the left.

You can imagine that graphing linear equations in three variables is quite involved and that such graphing is certainly beyond the capability of most hand-held graphing calculators. Therefore, an understanding of algebraic methods is critical to solving systems in three variables. Nevertheless, the visual model gives a fascinating interpretation to the solution of a system of three equations in three variables: the solution corresponds to the intersection of three surfaces, where each surface corresponds to one of the three equations.

Turn to page 41 of *MATHPOWER™ 11* and read “Solving Systems of Linear Equations in Three Variables” and “Explore: Use the Equations.”

Note: Equation 1 indicates that the sum of the major public galleries in Central Canada, Western Canada, and Eastern Canada is 28; equation 2 indicates that the sum of the number of galleries in Central Canada and Eastern Canada minus the number of galleries in Western Canada is 4; and, finally, equation 3 indicates that the number of galleries in Central Canada less twice the number of galleries in Eastern Canada plus the number of galleries in Western Canada is 10.

1. Answer questions 1 to 5, 6.b., and 7.b. of “Inquire” on page 41 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 5.

Recall that a solution to a linear system of two equations in x and y is an ordered pair, such as (3, 4). This ordered pair can be checked by substitution. Replace x with 3 and y with 4 in each equation. Both equations must be satisfied in order for the ordered pair to be a solution.

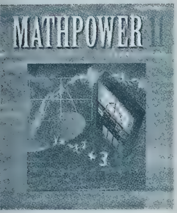
The solution to a linear system of three equations in x, y, and z is an ordered triple, such as (3, 4, 5). This ordered triple can also be checked by substitution. Replace x with 3, y with 4, and z with 5. Again, each equation must be satisfied in order for the ordered triple to be a solution.

Mathematical Process

☐ Communication
☒ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☒ Reasoning
☐ Technology
☐ Visualization

2. Turn to page 44 of *MATHPOWER™ 11* and do questions 1, 2, 6, 8, and 10 of “Practice.”

Compare your responses with those in the Appendix, Section 1: Activity 5.



Mathematical Process

- ■ Communication
- ☒ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization

Turn to page 41 of *MATHPOWER*™ *11* and read from the red line to the bottom of page 42, working through Example 1.

Notice that the first part of the solution is reducing the system from three equations in three variables to two equations in two variables. Once you obtain a system in two variables, you can continue the solution using the algebraic or graphical methods you studied earlier.

3. Answer the following questions on pages 44 and 45 of the textbook:

a. questions 12, 14, 16, and 26 of "Practice"
b. question 51 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 5.

For some systems of equations in three variables, you may prefer to use the substitution method to reduce the system. The substitution method works particularly well when one of the variables can easily be expressed in terms of the others. Look at the following system of equations:

$c+p+w=456$ ①
$c-p=80$ ②
$p=2w-2$ ③

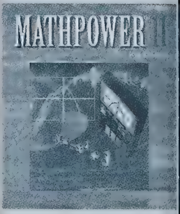
Replace p with $2w-2$ in both ① and ②.

$c+w+(2w-2)=456$ ⑤
$c-(2w-2)=80$ ⑥

This system reduces to

$c+3w=458$ ⑤
$c-2w=78$ ⑥

Now, this system can be solved easily by elimination or substitution.



Turn to page 43 of *MATHPOWER™ 11* and work through Example 2. Note how the substitution method is used rather than the elimination method.

4. Answer the following questions on pages 44 and 45 of the textbook:

- **a.** questions 27, 28, 30, 32, and 34 of "Practice"
- **b.** questions 41, 42, 43, and 49 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 5.

When solving systems of rational equations you can multiply any of the equations by a suitable constant in order to get rid of decimals or denominators.

Example

Simplify the following rational equations:

a. $\frac{2x}{5}+\frac{2y}{3}+\frac{7z}{10}=\frac{1}{5}$

b. $1.3a+1.09b-0.03c=5$

Solution

a.

$$\frac{2x}{5}+\frac{2y}{3}+\frac{7z}{10}=\frac{1}{5}$$

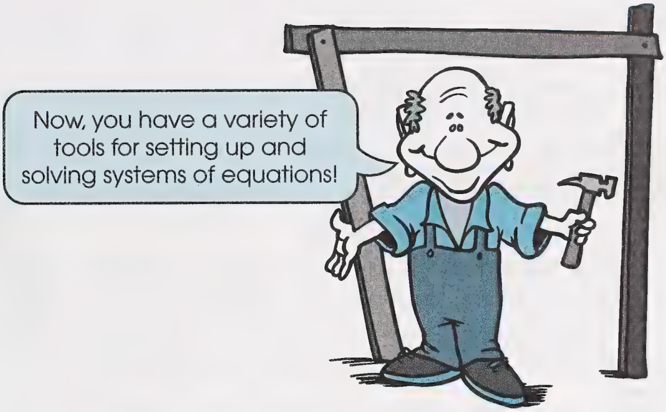
$$\left(\overset{6}{\cancel{30}}\right)\frac{2x}{\underset{1}{\cancel{5}}}+\left(\overset{10}{\cancel{30}}\right)\frac{2y}{\underset{1}{\cancel{3}}}+\left(\overset{3}{\cancel{30}}\right)\frac{7z}{\underset{1}{\cancel{10}}}=\left(\overset{6}{\cancel{30}}\right)\frac{1}{\underset{1}{\cancel{5}}} \quad \longleftarrow \text{Multiply both sides by 30, the LCD.}$$

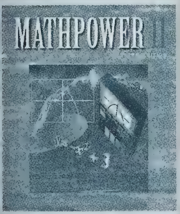
$$12x+20y+21z=6 \quad \longleftarrow \text{Simplify.}$$

b. $1.3a+1.09b-0.03c=5$

$$130a+109b-3c=500 \quad \longleftarrow \text{Multiply both sides by 100 to remove all decimal values.}$$

Once the coefficients of the equations in a system are integers, the system can be solved in the usual way.


Now, you have a variety of tools for setting up and solving systems of equations!



5. Turn to pages 44 to 46 of *MATHPOWER™ 11* and answer the following:

   a. questions 36 and 38 of "Practice"
   b. questions 52, 53, and 57 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 5.

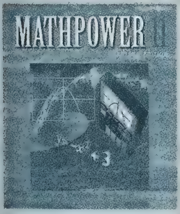
Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Now Try This

Scientists have long been interested in the fossil records of Primates, which include monkeys and the great apes. Did you know that the relative populations have changed over the millennia? In the world today, there are more monkeys than great apes.

A system of equations can be used to model the changing relationship between the great ape and monkey populations.



Turn to "CONNECTING MATH AND ZOOLOGY" on page 50 of *MATHPOWER™ 11* and read "Ape/Monkey Populations."

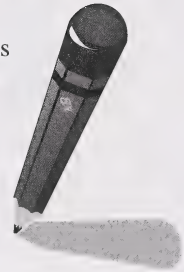
6. Answer the following questions on page 51 of the textbook:

   a. questions 1 to 3 of Investigation 1, "Interpreting the Graph"
   b. questions 1 and 2 of Investigation 2, "Solving Algebraically"

Compare your responses with those in the Appendix, Section 1: Activity 5.

Looking Back

In this activity, you generalized your understanding of linear systems from equations in two variables to solving systems of equations in three variables. You then used these systems to solve more-complex problems and discovered that the solutions to these problems were ordered triples of real numbers.



In your journal, write a system of three equations so that no equations are equivalent, each equation has three variables, and the solution to the system is (1, 2, 3). Compare your equations with those of another student taking Pure Mathematics 20. Are your equations the same? Why or why not?

Mathematical Process

- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

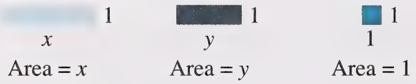
Extra Help

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

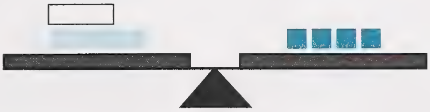
Your understanding of systems of equations and their solutions may be enhanced by modeling them with algebra tiles.

In this Student Module Booklet, white tiles are used to represent negative variables and numbers and coloured tiles are used to represent positive variables and numbers. The tiles used in this Student Module Booklet differ in colour from those used in the textbook. All of the positive tiles you will be using throughout this module are as follows:



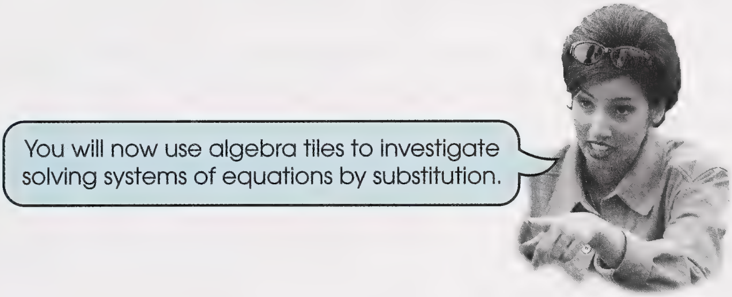
Example

Write the equation represented by the balance scale.

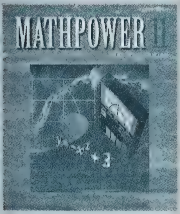


Solution

On the left side of the balance scale, there is one x-tile and one $-y$-tile. On the right side of the balance scale, there are four 1-tiles. Therefore, the equation is $x - y = 4$.


You will now use algebra tiles to investigate solving systems of equations by substitution.

Substitution

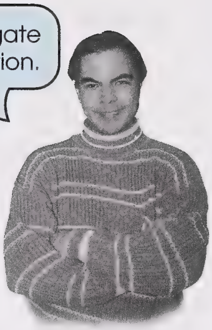


Turn to “INVESTIGATING MATH” on pages 18 to 20 of *MATHPOWER™ 11* and answer the following:

1. question 2 of Investigation 2, “Representing Systems of Equations”
2. question 2 of Investigation 3, “Solving Systems by Substitution, I”
3. question 2 of Investigation 4, “Solving Systems by Substitution, II”

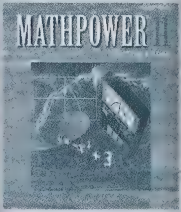
Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Extra Help.

Now, you will use algebra tiles to investigate solving systems of equations by elimination.



Elimination

Turn to "INVESTIGATING MATH" on pages 32 and 33 of *MATHPOWER™ 11* and read Investigation 1, "Solving Systems by Addition" and Investigation 2, "Solving Systems by Subtraction."



4. Answer the following questions on pages 32 and 33 of the textbook:

- **a.** question 1 of Investigation 1, "Solving Systems by Addition"
- **b.** question 2 of Investigation 2, "Solving Systems by Subtraction"

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Extra Help.

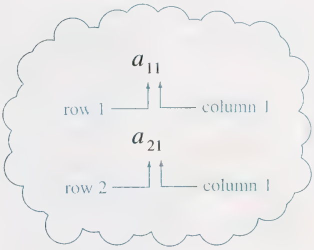
Enrichment

You probably found that solving systems of equations using the substitution method or the elimination method involves considerable algebraic manipulation. As a result, it is easy to make an error. So, a solution in which your calculator does some of the algebraic work may seem pretty attractive. That's what this Enrichment is about.

This calculator method involves entering the coefficients from the system as a rectangular array of numbers. Such a rectangular array is called a **matrix**.

matrix: an arrangement of numbers in rows and columns where the position of each number is of mathematical significance

An example of a 2×2 matrix is $\begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$. This matrix is of the form $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.



When solving a system of equations, the system is first rewritten as follows:

$$a_{11}x + a_{12}y = c_1$$
$$a_{21}x + a_{22}y = c_2$$

As you can see, the terms containing the variables are placed on the left side of the equations and the constants are placed on the right side of the equations.

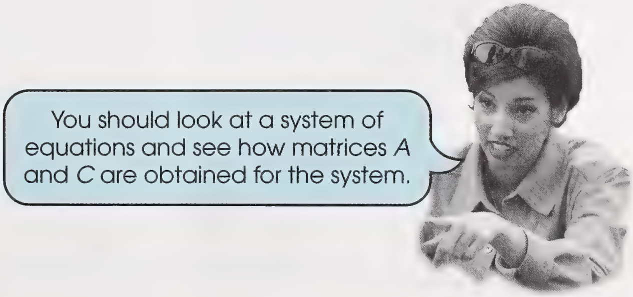
The coefficients are then written as a matrix—a square one.

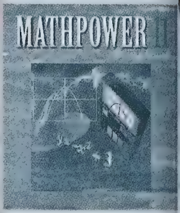
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

This matrix of coefficients is often represented by A.

The variables are represented by the matrix $\begin{bmatrix} x \\ y \end{bmatrix}$, which can be symbolized by X; and the constants are represented by the matrix $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, which can be symbolized by C.

Now, you can express the system of equations as $A \times X = C$—a single matrix equation!


You should look at a system of equations and see how matrices *A* and *C* are obtained for the system.



Turn to page 22 of *MATHPOWER™ 11* and refer to Example 1. This system of equations will be used to demonstrate solving by matrices.

$x + 4y = 6$ ①
$2x - 3y = 1$ ②

The system is already organized for a matrix solution. Matrix *A* for this system is formed from the coefficients of the variables.

$$\begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}$$

Matrix *C* is formed from the coefficients to the right of the equal signs.

$$\begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

In the equation $A \times X = C$, the multiplication symbol, ×, stands for a special operation, called **matrix multiplication**. Matrix multiplication is not the familiar kind of multiplication; but, there are some parallels you should be aware of. Fortunately, much of the detail of this new operation can be left to your calculator.

Suppose you have the algebraic equation $6w = 24$. You know that the value of *w* can be expressed as $w = \frac{1}{6} \times 24 = 4$. The important observation is that the value of *w* is the multiplicative inverse of its original coefficient times the number originally to the right of the equal sign.

So, for the matrix equation, the solution to the matrix equation $A \times X = C$ is $X = A^{-1} \times C$. This can be written as $X = A^{-1}C$, where A^{-1} is the multiplicative inverse of *A*. You will see that the value for matrix *X* will reveal the solution to the system of equations with which you started.

Return to the original system of equations.

$x + 4y = 6$ ①
$2x - 3y = 1$ ②

For this system, $A = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}$ and $C = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$. Based on these matrices, $X = A^{-1}C$.

Mathematical Process

☐ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☒ Technology
☐ Visualization

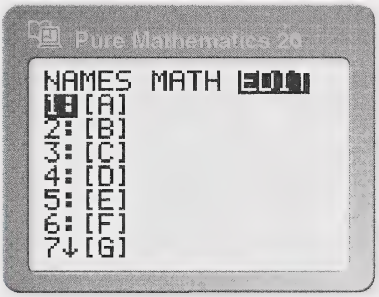
Now, it's time to use your graphing calculator to determine the matrix value of *X*, the solution of the equation.

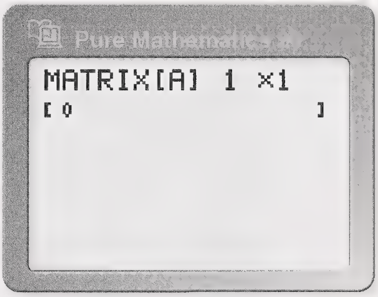
Step 1: Write the system of equations with the variables on the left and in corresponding positions and with the constants on the right.

$x+4y=6$ ①
$2x-3y=1$ ②

Step 2: Enter matrix *A*.

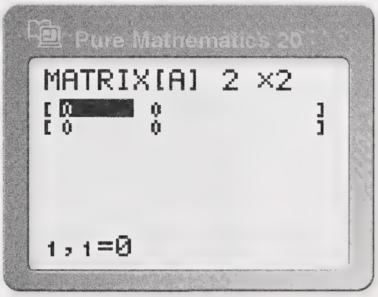
- Press MATRX, select the EDIT menu, and press 1 to select matrix *A*.





```
MATRIX[A] 1 ×1
[0                  ]
```

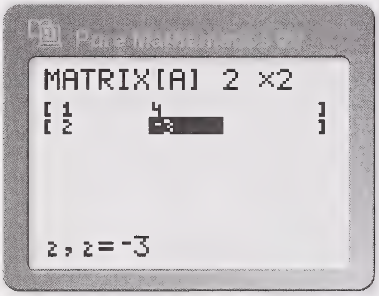
- Change the expression "1×1" at the top of the screen to "2×2." This indicates that there are 2 rows and 2 columns in this matrix.



```
MATRIX[A] 2 ×2
[0       0          ]
[0       0          ]

1,1=0
```

- Enter the elements of the matrix going across the first row and then across the second row.

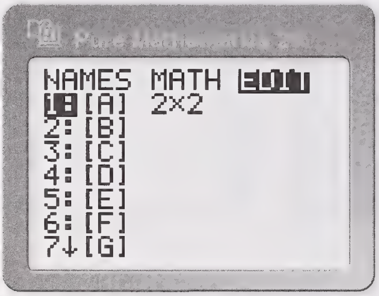
1 ENTER 4 ENTER 2 ENTER (–) 3 ENTER

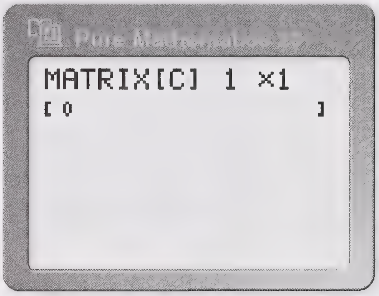


```
MATRIX[A] 2 ×2
[ 1       4          ]
[ 2       -3         ]

2,2=-3
```

Step 3: Enter matrix *C*.

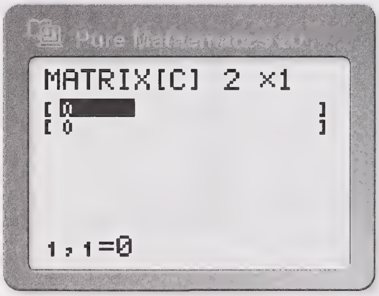
- Press MATRX, select the EDIT menu, and press 3 to select matrix *C*.





```
MATRIX[C] 1 ×1
[ 0                  ]
```

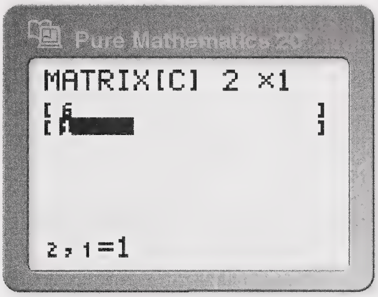
- Change the expression "1×1" at the top of the screen to "2×1." This indicates that there are 2 rows and 1 column in this matrix.



```
MATRIX[C] 2 ×1
[ 0                  ]
[ 0                  ]

1,1=0
```

- Enter the two elements of the matrix.

6 ENTER 1 ENTER

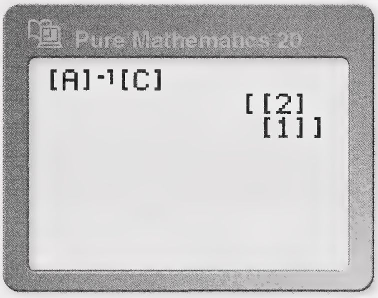


```
MATRIX[C] 2 ×1
[6         ]
[1         ]

2,1=1
```

- Press 2nd [QUIT] to quit the entry of C.

Step 4: Determine the value of matrix X by calculating $A^{-1}C$.

MATRX 1 x^{-1} MATRX 3 ENTER



```
[A]⁻¹[C]
              [[2]
               [1]]
```

Because matrix X is $\begin{bmatrix} x \\ y \end{bmatrix}$, the calculator shows that this matrix is equal to $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Therefore, $x = 2$ and $y = 1$.

Check

LS	RS
$x+4y$	6
$=(2)+4(1)$	
$=6$	

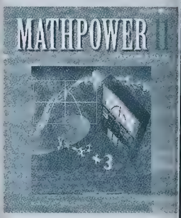
LS = RS

LS	RS
$2x-3y$	1
$=2(2)-3(1)$	
$=1$	

LS = RS

The solution is $(2,1)$.

1. Turn to page 39 of *MATHPOWER*™ *11* and answer questions 18, 35, and 36 of “Practice” using matrices on your graphing calculator.



Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Enrichment.

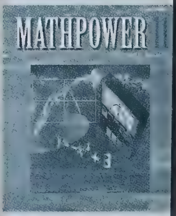
In the preceding questions, you solved systems of two equations in two variables using matrices. Larger systems can also be solved using matrices. For a system of three equations in three variables, the matrix of the coefficients, *A*, has three rows and three columns. The matrix of constants, *C*, has three rows and one column.

Remember: You will have to alter the dimensions of the matrices when working with larger systems.

Mathematical Process

■ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
☒ Technology
■ Visualization

2. Use a graphing calculator to answer the following:
   a. Turn to page 42 of *MATHPOWER*™ *11* and solve the system of equations in Example 1 using matrices.
   b. Turn to page 46 of *MATHPOWER*™ *11* and answer question 57 of “Applications and Problem Solving” using matrices.



Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Enrichment.

Conclusion

In this section, you reviewed how linear equations can be used to model and solve real-world problems. You discovered that some real-world problems are best represented by a system of two linear equations in two variables. You explored solving these systems by graphing, by substitution, and by elimination. Finally, you extended your understanding of systems of equations by solving linear systems consisting of three equations with three unknowns.

In the introduction to this section, you solved a problem involving World Heritage Sites using systems of linear equations. One of the most spectacular heritage sites in the world is the Great Wall of China. Did you know that the Great Wall of China can be seen with the naked eye from orbiting space stations?

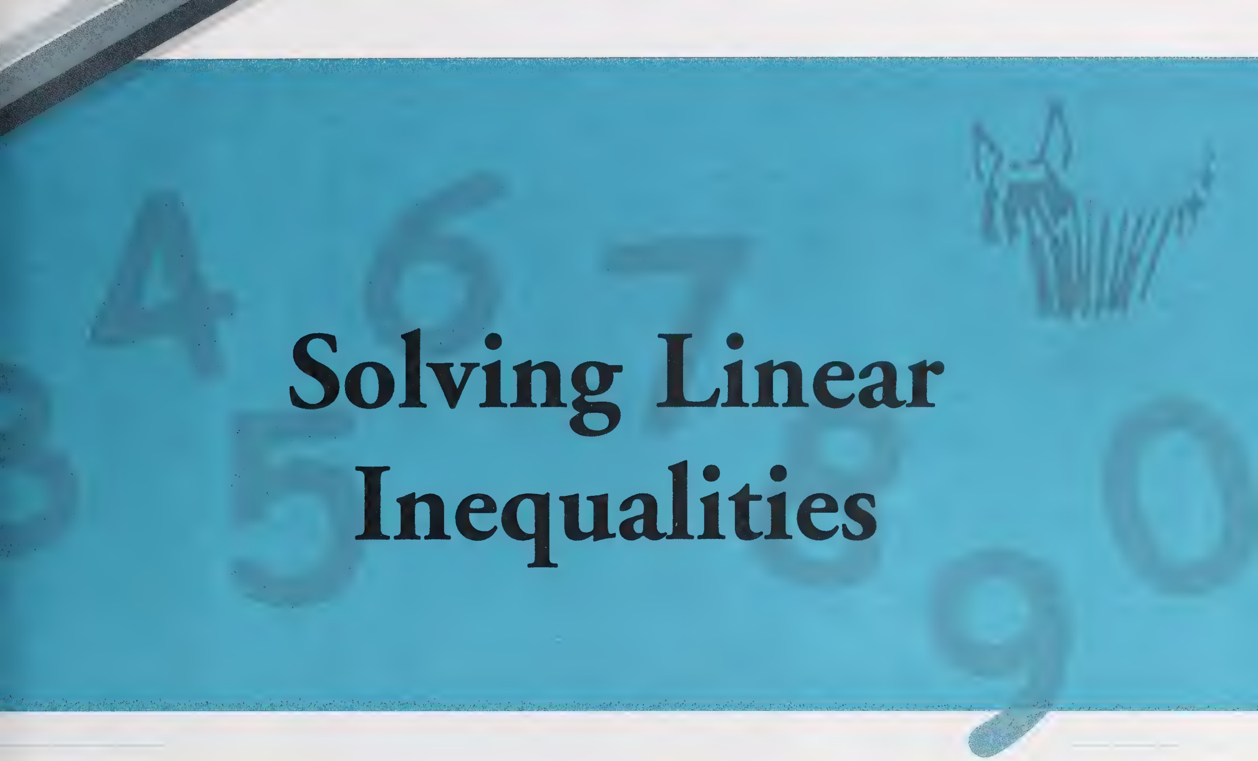
If you have access to the Internet, you can explore any World Heritage Site and its location by visiting the following website:

http://www.its.caltech.edu/~salmon/world.heritage.html

Assignment

You are now ready to complete the assignment for Section 1.

Section 2



Solving Linear Inequalities

Have you been fortunate enough to travel internationally? Such travel likely involved dealing with an airline. When you travel by car you can take a lot of gear with you; but when you travel by plane, you have to pay close attention to what you pack.

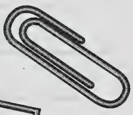
Airlines impose certain luggage and weight restrictions. For example, you may be restricted to two pieces of checked luggage. These pieces must each weigh no more than 32 kg. The sum of the dimensions of the first piece must be 158 cm or less. The sum the dimensions of the second piece must be 140 cm or less.

You can represent the size restrictions of the first piece of checked luggage by the following linear inequality, where ℓ represents the length, w represents the width, and h represents the height.

$$\ell + w + h \leq 158$$

What would be the linear inequality for the second piece of checked luggage?

In this section, you will solve and graph inequalities that are in one variable and in two variables, apply inequalities to solving problems, and extend your general problem-solving skills.



Activity 1: Inequalities in One Variable

Many statements are expressed in terms of equality: the distance from Edmonton to Vancouver is 817 km as the crow flies; the height of the Petronas Towers in Kuala Lumpur, Malaysia, is 450 m, making these the tallest buildings in the world (as of 1999); and the charge of an electron equals 1.6×10^{-19} coulombs.

Statements of inequality can also be useful in describing situations. For example, think about the achievements of a speed skater. A speed skater breaks a current record by completing a race in less time than ever recorded for that distance.

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

In this activity, you will review the techniques for solving and graphing the solutions to inequalities in one variable.

Turn to page 60 of *MATHPOWER™ 11* and read "Reviewing Linear Inequalities in One Variable."

An inequality may be sufficient to describe many situations, but sometimes a more limited range of values must be indicated. This can be done using a **compound inequality**.

compound inequality: a compound statement expressing two inequalities at the same time

Turn to "GETTING STARTED" on page 58 of *MATHPOWER™ 11* and read "Frequency Ranges."

1. Answer question 1 of "Frequency Ranges" on page 58 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 1.

Remember: When graphing inequalities, a closed dot, indicating inclusion, corresponds to *greater than or equal to* or *less than or equal to.* An open dot, indicating exclusion, corresponds to *greater than* or *less than*.

2. Turn to page 59 of *MATHPOWER™ 11* and answer questions 15, 16, 19, 20, and 24 of "Warm Up."

Compare our responses with those in the Appendix, Section 2: Activity 1.

Before landing, an airplane's rate of descent must lie within a certain range. Too fast may lead to an abrupt, unsafe landing; too slow may mean not enough speed to maintain lift. Shortly, you will use inequalities to determine the range of speed for a safe descent.

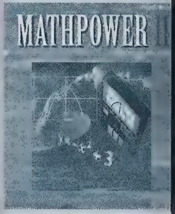
3. Turn to pages 60 and 61 of *MATHPOWER™ 11* and answer the following:

- **a.** questions a. and b. of "Explore: Solve the Inequalities"
- **b.** questions 1, 2, 4.b., 4.d., 5, and 6 of "Inquire"

Compare your responses with those in the Appendix, Section 2: Activity 1.

Pay special attention when you multiply or divide both sides of an inequality by a negative value.

Solving with Algebra



Turn to page 61 of *MATHPOWER™ 11*, and read from the red line near the top of the page to the bottom of the page, working through Examples 1 and 2.

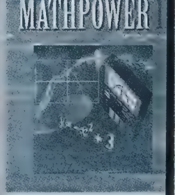
4. Answer the following questions on pages 63 and 64 of the textbook:

- **a.** questions 12, 16, 26, 30, 34, and 36 of "Practice"
- **b.** questions 51 to 54, 56, and 60 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 1.

Some inequalities involve more than one fraction. To solve these, multiply each term by the lowest common denominator (LCD) to simplify the inequality.

Likewise, you can simplify inequalities having decimal values by multiplying both sides of the inequality by a suitable power of 10 that will clear all decimal values.



Turn to page 62 of *MATHPOWER™ 11* and work through Example 3.

5. Answer questions 39, 45, 48, and 50 of "Practice" on page 63 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematical Process

- ■ Communication
- ■ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ☒ Technology
- ☒ Visualization

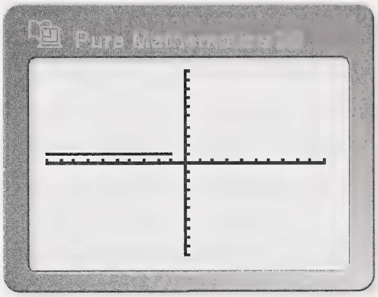
Solving with a Graphing Calculator

Sometimes, you may want to go directly from a given inequality to its graph without going through the algebra. In these situations, you can use your graphing calculator.

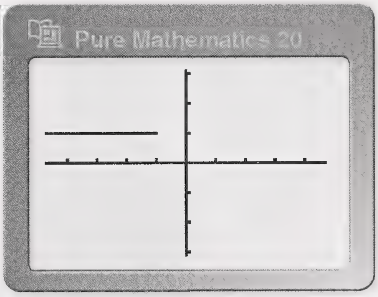
Suppose you want to graph $3(x+5)+2(x-9)\le -8$ on your graphing calculator.

First, make sure the calculator is in Dot mode; then press the following:

Y= 3 (X,T,Θ,*n* + 5)
+ 2 (X,T,Θ,*n* − 9)
2nd [TEST] 6 (−) 8 GRAPH



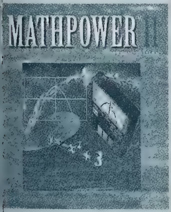
To see the graph more clearly, press ZOOM 4 (4:ZDecimal). Your display should appear like the one on the right.



The graphing calculator displays the graphs of inequalities just above the *x*-axis rather than directly on the *x*-axis. This allows the graph to be clearly visible. For each *x*-value satisfying the inequality, a point with a *y*-value of 1 is displayed. For each *x*-value not satisfying the inequality, the *y*-value is set to 0 and the point is not visible on the screen. The graph does not indicate whether the interval is open or closed. Instead, you must use the Trace feature to determine if boundary point -1 is a solution to the inequality.

6. Turn to "TECHNOLOGY" on page 65 of *MATHPOWER*™ *11* and answer the following:

a. questions 1, 3, and 4 of Investigation 1, "Displaying a Solution"
b. questions 5, 9, and 13 of Investigation 2, "Solving Inequalities"



Compare your responses with those in the Appendix, Section 2: Activity 1.

When you use your graphing calculator to graph inequalities in one variable, the graphs are displayed one unit above the x-axis. Remember, the solution is only the set of x-values of these points. Next, you will discover when you use the entire coordinate plane to display inequalities

Solutions and the Coordinate Plane

When the solution of an inequality is made up of ordered pairs, you must use the coordinate plane to graph the solution.

Consider the ordered pairs (x, y) that satisfy the inequality $x \le 2$. Think of some ordered pairs that fit, such as $(2, 1)$, $(-3, 0)$, $(0, 10)$, $(0, 0)$, and $(1, -9)$. In these ordered pairs, it does not matter what the y-value is; the inequality does not even refer to the y-value. The x-value, however, must be less than or equal to 2 in order for the ordered pair to satisfy the inequality.

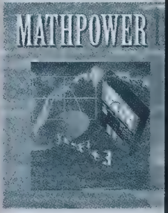
Similarly, for an inequality expressed only in terms of y, such as $y \le 2$, no restrictions are imposed on the x-value of ordered pairs. Only the y-value must conform to the condition.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☐ Visualization

Now, do some investigation on your own to discover what the solutions of inequalities look like on the coordinate plane.

Turn to "INVESTIGATING MATH" on page 68 of *MATHPOWER*™ *11* and read "Graphing Inequalities in the Coordinate Plane."



7. Answer questions 1, 2, 3, 4.d., 4.e., 4.f., 5, 6.b., 6.c., 7, 8.b., and 8.c. of Investigation 1, "Inequalities in One Variable" on pages 68 and 69.

Compare your responses with those in the Appendix, Section 2: Activity 1.

---

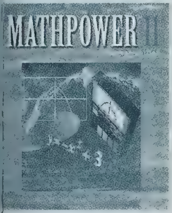
boundary line: a li
on the coordinate
plane that separates
the plane into two
regions

As you have seen, once you have the **boundary line** placed in the coordinate plane, you can complete graphing the inequality by indicating whether or not the boundary line is included and then by shading the appropriate region (or regions).

Now Try This

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☒ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization



By developing problem-solving skills, you can make your thinking more effective. One way to approach a given problem is by asking yourself a question that is a simpler version of the problem. Often, the answer to the question you posed sheds light on the more-complex problem.

Turn to "PROBLEM SOLVING" on page 66 of *MATHPOWER™ 11* and read "Solving a Simpler Problem."

8. Answer questions 1, 2, 3, 6, 9, and 11 of "Applications and Problem Solving" on page 67 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 1.

Looking Back

Mathematical Process

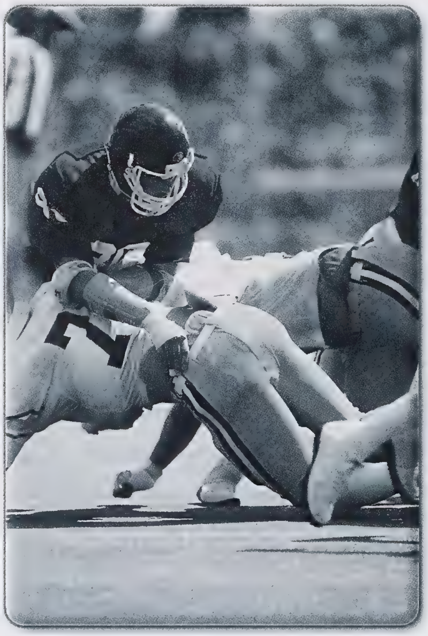
- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

In this activity, you described conditions using inequalities in one variable, graphed inequalities on the number line and in the coordinate plane, solved inequalities, and applied inequalities to solve real-world situations. Following this, you gained experience in using simpler questions to solve challenging problems.

In your journal, compare the way you solve an equation to the way you solve an inequality.

Activity 2: Inequalities in Two Variables

Inequalities containing two variables may be necessary to represent certain situations that cannot be described by inequalities in one variable. An inequality containing two variables can address two quantities at once.



For example, an inequality in two variables can be used to describe the relationship between the number of wins and losses of a football team during a regular season.

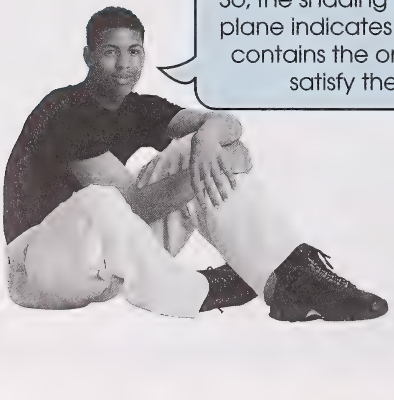
Turn to page 72 of *MATHPOWER™ 11* and read "Graphing Linear Inequalities in Two Variables."

As is the case for equations in two variables, the solution to an inequality in two variables is composed of ordered pairs and, thus, must be graphed in the coordinate plane.

1. Answer the following questions on page 72 of the textbook:

- **a.** questions a. and b. of "Explore: Interpret the Data"
- **b.** questions 1 to 6, 7.a., and 7.c. of "Inquire"

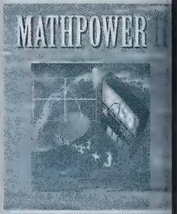
Compare your responses with those in the Appendix, Section 2: Activity 2.



So, the shading in the coordinate plane indicates which **half-plane** contains the ordered pairs that satisfy the inequality.

Mathematical Process

- ■ Communication
- ☒ Connection
- ■ Estimation
- ■ Mental Math
- ■ Problem Solving
- ■ Reasoning
- ■ Technology
- ■ Visualization



half-plane: one of the two regions on the coordinate plane separated by a boundary line

athematical
rocess

☐ Communication
☐ Connection
☐ Estimation
☑ Mental Math
☐ Problem Solving
☐ Reasoning
☐ Technology
☐ Visualization

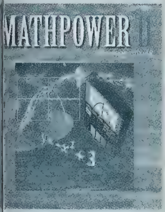
In this activity, substituting ordered pairs in linear inequalities to check whether you have shaded the correct half of the coordinate plane is an important skill. Being able to do this mentally will save you time and reduce the written portion of your solution to the essentials. The following set of questions will allow you to practise substituting ordered pairs mentally.

2. Turn to page 59 of *MATHPOWER™ 11* and answer questions 7 to 12 of "Mental Math: Equations and Inequalities."

Compare your responses with those in the Appendix, Section 2: Activity 2.

You will now build on your initial understandings of inequalities in two variables.

Solving and Graphing Inequalities in Two Variables



Turn to page 72 of *MATHPOWER™ 11* and read from the red line to the bottom of the page.

Did you notice that the direction of the inequality sign indicates which half-plane is part of the graph when y is isolated on one side of the inequality?

Remember: Solve the inequality for y so you can use the direction of the inequality symbol to determine which region to shade.

In the following examples, pay special attention to how to determine which half-plane is shaded and whether or not the boundary line is included.

Turn to pages 73 and 74 of *MATHPOWER™ 11* and work through Examples 1 and 2.

3. Answer questions 3, 4, 7, 8, 11, 14, 15, 17, 22, 27, and 28 of "Practice" on page 75 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 2.

Restricting the Variables

So far, the graphs of inequalities you have encountered in this activity are half-planes with or without their boundary lines. However, there may be constraints on the variables of an inequality, limiting the graphs to a smaller part of the coordinate plane.

Take the time to discover for yourself the nature of inequalities with restricted variables.

Turn to page 70 of *MATHPOWER™ 11* and read Investigation 3, "Restricting the Variables."

4. Answer questions 1, 2, 3, 4.b., 4.d., 4.f., and 7.a. of Investigation 3, "Restricting the Variables."

Compare your responses with those in the Appendix, Section 2: Activity 2.

As you will see in the next example, when inequalities are used to represent real-world situations, there are likely to be restrictions on the variables. For example, numbers of objects are normally non-negative integers.

Turn to page 74 of *MATHPOWER™ 11* and work through Example 3.

5. Answer the following questions on pages 75 and 76 of the textbook:

- **a.** questions 31 and 34 of “Practice”
- **b.** questions 35, 40.a., and 42 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 2: Activity 2.

Using a Graphing Calculator

athematical rocess

☐ Communication
☐ Connection
☐ Estimation
☐ Mental Math
☐ Problem Solving
☐ Reasoning
☒ Technology
☐ Visualization

You can use your graphing calculator to graph inequalities in two variables. The inequality is not entered directly; it is the equation of the boundary line that is entered. Then you will have to select the region (or half-plane), above or below the boundary line, that is to be shaded.

CAUTION

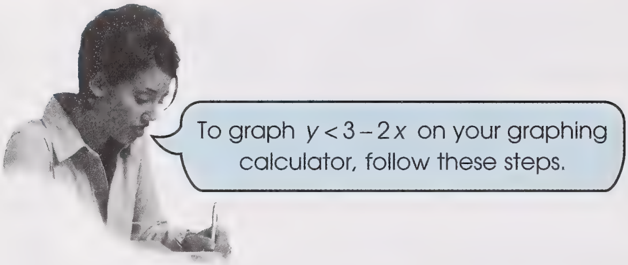
The graphing calculator does not show whether the boundary line is included or excluded.

First, convert the inequality to slope and y-intercept form, y [illegible] $mx+b$. Where, [illegible] is an inequality symbol.

Next, graph the corresponding equation, $y=mx+b$, and determine whether the shading should be above or below this line. **Note:** The order of the terms on the right side of the inequality does not matter.

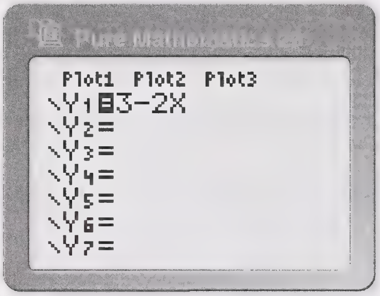
Turn to “TECHNOLOGY” on page 71 of *MATHPOWER™ 11* and work through the method shown in Investigation 2, “Solving for y and Graphing.”

Remember: Solve the inequality for y so you can use the direction of the inequality symbol to determine which region to shade.


To graph $y<3-2x$ on your graphing calculator, follow these steps.

Step 1: Enter the corresponding equation, $y=3-2x$.

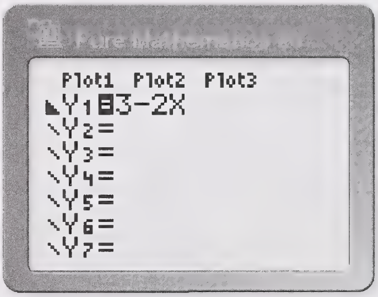
Y= 3 − 2 X,T,Θ,*n*



```
Plot1  Plot2  Plot3
\Y1=3-2X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

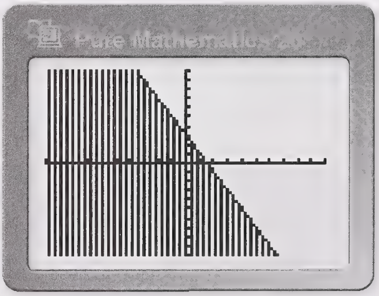
Step 2: Enter which half-plane is to be shaded.

Move the cursor to the left of "Y_1" using the arrow keys, and press ENTER repeatedly to cycle through the graph-style icons, stopping at the Below icon. (Refer to the display.)

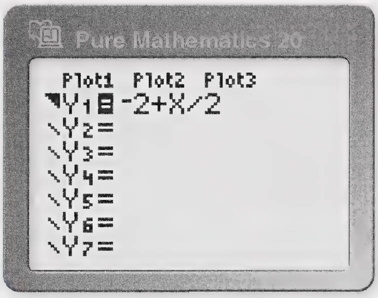


```
Plot1  Plot2  Plot3
◣Y1=3-2X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

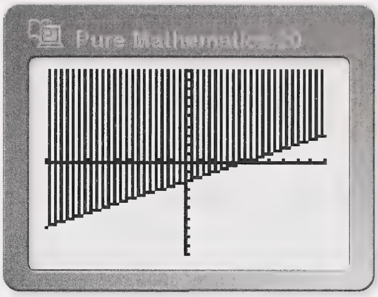
Step 3: Press GRAPH to display the inequality.

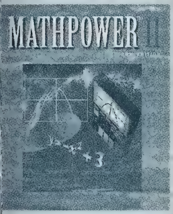


The procedure for graphing the inequality $y \geq -2 + \frac{x}{2}$ on your calculator is similar.



```
Plot1  Plot2  Plot3
▼Y1▣-2+X/2
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```





6. Turn to "TECHNOLOGY" on page 71 of *MATHPOWER™ 11* and answer the following:

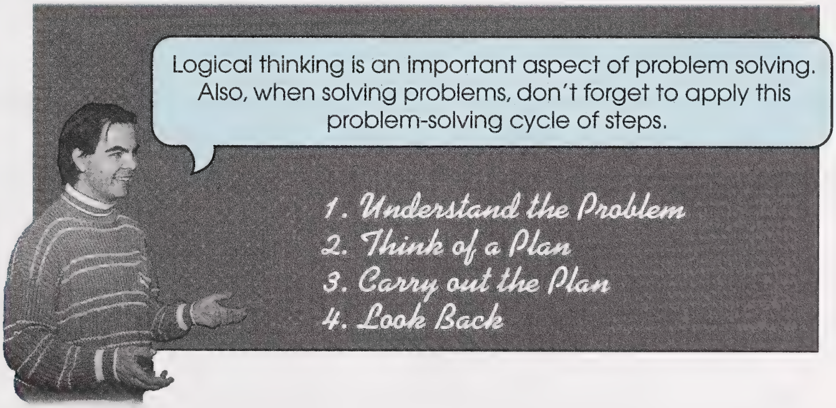
   a. question 11 of Investigation 1, "Graphing Inequalities"
   b. questions 11 and 13 of Investigation 2, "Solving for y and Graphing"

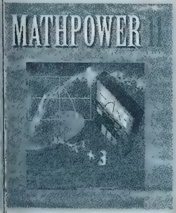
Compare your responses with those in the Appendix, Section 2: Activity 2.

Now Try This

Mathematical Process

- ■ Communication
- ☒ Connection
- ■ Estimation
- ■ Mental Math
- ☒ Problem Solving
- ☒ Reasoning
- ■ Technology
- ☒ Visualization


Logical thinking is an important aspect of problem solving. Also, when solving problems, don't forget to apply this problem-solving cycle of steps.
1. Understand the Problem
2. Think of a Plan
3. Carry out the Plan
4. Look Back



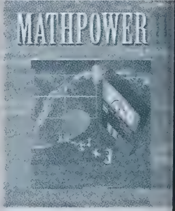
Turn to "PROBLEM SOLVING" on page 78 of *MATHPOWER™ 11* and read "Use Logic."

If you had difficulty accepting the fact that Justine is the same distance from her farm at exactly the same time on the two days, then you may want to think of a simpler situation.

One day, Justine left her farm at 08:00 to go to her friend's house in town. Meanwhile, Justine's friend left her house in town at 08:00 to go to Justine's farm. They both follow the same road and arrive at their separate destinations at the same time, 15:00. Show that, at one point, somewhere on the road, they are both the same distance from Justine's farm at exactly the same time, no matter at what speed each had travelled.

When you consider the travellers taking the trip during the same time period, you can easily see that the friends must meet and pass each other at some time between 08:00 and 15:00. At that one point on the road where they meet, they are indeed the same distance from the farm (or anything else for that matter).

7. Turn to page 79 of *MATHPOWER™ 11* and answer question 1 of "Applications and Problem Solving."



Compare your response with the one in the Appendix, Section 2: Activity 2.

Looking Back

In this activity, you solved and graphed linear inequalities in two variables using algebraic techniques and your graphing calculator. You also graphed inequalities that had constraints on the variables. You then gained more experience using logical thinking to solve problems.

In your journal, describe the graphs of the following inequalities. Use diagrams to illustrate your descriptions.

- $y > mx + b$
- $y < mx + b$
- $y > b$
- $y < b$

Mathematical Process

☒ Communication
■ Connection
■ Estimation
■ Mental Math
■ Problem Solving
■ Reasoning
■ Technology
■ Visualization

Follow-up Activities

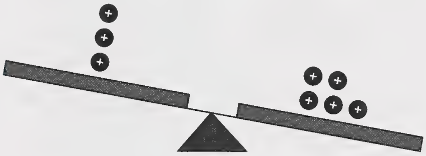
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Many students forget to reverse the inequality sign when multiplying or dividing by a negative number. Has this happened to you? Modelling the effects of multiplication on an inequality may help you to remember. Here are two examples.

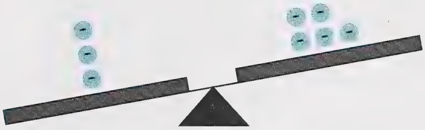
Example 1

The inequality $3 < 5$ can be modelled as follows:



$3 < 5$

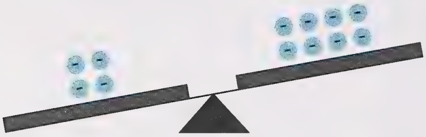
If you multiply each side by -1 (or replace each side with its inverse), the inequality changes.



$-3 > -5$

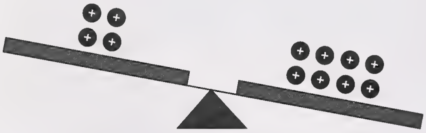
Example 2

The inequality $-4 > -8$ can be modelled as follows:



$-4 > -8$

If you multiply each side by -1 (or replace each side with its inverse), the inequality changes.

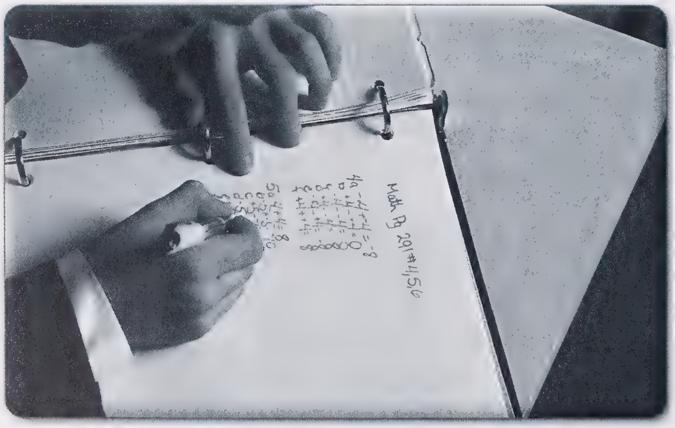


$4 < 8$

Solve and check each of the following inequalities; then graph your solution on a number line.

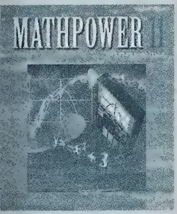
1. $-3x \le 6$ **2.** $-4x + 4 > 8$

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Extra Help.



Enrichment

In Section 1, you solved systems of equations. In this section, you focussed on inequalities. In this Enrichment, you will solve systems of inequalities.



Turn to page 80 of *MATHPOWER™ 11* and read "Solving Systems of Linear Inequalities."

1. Answer the following questions on page 80 of the textbook:

- **a.** questions a. and b. of "Explore: Draw a Graph"
- **b.** questions 1, 3, and 4 of "Inquire"

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Enrichment.

You have seen how a system of inequalities can effectively represent the performance requirements from a hockey team during the playoffs. Now, turn to page 80 of *MATHPOWER™ 11* and read from the red line to the bottom of page 81, working through Example 1.

2. Turn to page 85 of *MATHPOWER™ 11* and answer questions 7 to 9 of "Practice."

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Enrichment.

Conclusion

In this section, you began by exploring inequalities in one variable. You used these inequalities to model real-world problems, and you graphed their solutions on a number line. You then extended these basic techniques to more-complex situations modelled by inequalities in two variables. You solved these inequalities and graphed your solutions in the coordinate plane, using both pencil-and-paper and graphing calculator techniques.

As you discovered, the words "no more than" or "no less than" that appear in problem situations are often represented using inequalities. Limits on the size or weight of luggage you can take on a commercial flight, the number of people that can safely ride on an elevator, and the load limits on highways and bridges are a few examples of how inequalities are an integral part of everyday life.

Be sure to do "no less than" your best on the Section 2 assignment!

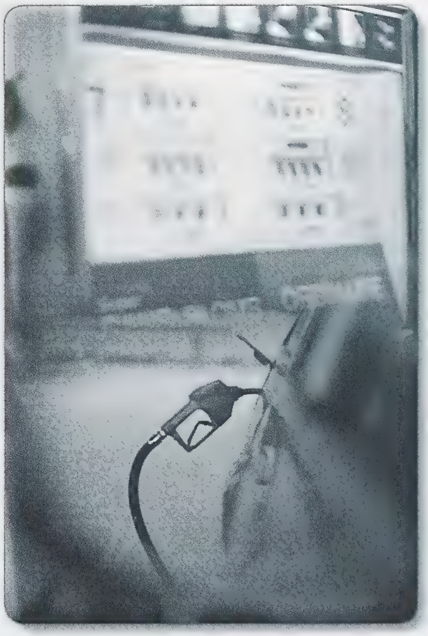
Assignment

You are now ready to complete the assignment for Section 2.

Module Summary

Throughout this module, you explored both equations and inequalities.

In Section 1, you reviewed solving equations containing a single variable and graphing linear equations in two variables; investigated systems of linear equations in two and three variables; and solved systems of linear equations by graphing, substitution, and elimination. Throughout this section, you applied your skills to real-world problems. Not only did you master pencil-and-paper techniques, you also used your graphing calculator to solve systems of equations.



In Section 2, you discovered that some problem situations are best modelled by inequalities. Once again, you reviewed techniques for solving problems involving one variable; then you extended these techniques to solving problems involving two variables.

It is difficult to think of problems that do not contain equations or inequalities. Have you or your friends ever had the problem of running low on fuel but only having just a few dollars to put in the tank? The farthest you can drive on the money you have can be a painful lesson of the practical application of inequalities.

You are now ready to complete the Final Module Assignment.

Appendix

Glossary
Suggested Answers

Glossary

algebraic expression: an expression containing only algebraic symbols and operations—addition, subtraction, multiplication, division, roots, and powers

boundary line: a line on the coordinate plane that separates the plane into two regions

compound inequality: a compound statement expressing two inequalities at the same time

cylindrical helix: a curve ascending a cylinder at a constant angle

half-plane: one of the two regions on the coordinate plane separated by a boundary line

matrix: an arrangement of numbers in rows and columns where the position of each number is of mathematical significance

system of equations: a group of equations that are to be considered at the same time

Suggested Answers

Section 1: Activity 1

1. Textbook questions 1, 3, 6, 8, and 10 of "Warm Up," p. 3

1. $$\begin{aligned}3x+2(1-x)&=3x+2-2x\\&=x+2\end{aligned}$$

3. $$\begin{aligned}2y-(5-y)&=2y-5+y\\&=3y-5\end{aligned}$$

6. $$\begin{aligned}(3c-4d)-(5c-4d)&=3c-4d-5c+4d\\&=-2c\end{aligned}$$

8. $$\begin{aligned}5m-2n+5(n-m)&=5m-2n+5n-5m\\&=3n\end{aligned}$$

10. $$\begin{aligned}p-2q+3r+2(p+q-2r)&=p-2q+3r+2p+2q-4r\\&=3p-r\end{aligned}$$

2. Textbook questions 19, 21, 23, and 24 of “Warm Up,” p. 3

19.

$$\frac{t-1}{3}=5-t$$

$$3\left(\frac{t-1}{3}\right)=3(5-t) \quad \longleftarrow \text{Multiply both sides by 3.}$$

$$t-1=15-3t$$

$$t=16-3t$$

$$4t=16$$

$$t=4$$

21.

$$\frac{x}{2}+\frac{1}{3}=-\frac{1}{2}$$

$$6\left(\frac{x}{2}+\frac{1}{3}\right)=6\left(-\frac{1}{2}\right) \quad \longleftarrow \text{Multiply both sides by 6, the LCD.}$$

$$3x+2=-3$$

$$3x=-5$$

$$x=-\frac{5}{3}$$

23. $3(x+3)=-2(2x-1)$

$$3x+9=-4x+2$$

$$3x=-4x-7$$

$$7x=-7$$

$$x=-1$$

24. Method 1

$$0.2(4n+1)=0.5(n-2)$$

$$0.8n+0.2=0.5n-1$$

$$0.8n=0.5n-1.2$$

$$0.3n=-1.2$$

$$n=-4$$

Method 2

Clear the decimals; then solve.

$$0.2(4n+1)=0.5(n-2)$$

$$10\times0.2(4n+1)=10\times0.5(n-2) \quad \longleftarrow \text{Multiply both sides by 10.}$$

$$2(4n+1)=5(n-2)$$

$$8n+2=5n-10$$

$$8n=5n-12$$

$$3n=-12$$

$$n=-4$$

Section 1: Activity 1 (continued)

3. Textbook questions 25, 27, 29, and 31 of "Warm Up," p. 3

25.
$$\begin{aligned} x+3y&=11 \\ x&=11-3y \end{aligned}$$

27.
$$\begin{aligned} x-2y+4&=0 \\ x&=2y-4 \end{aligned}$$

29.
$$\begin{aligned} 2x+y&=3 \\ y&=3-2x \end{aligned}$$

31.
$$\begin{aligned} 2x+4y&=-1 \\ 4y&=-2x-1 \\ y&=\frac{-2x-1}{4} \end{aligned}$$

4. a. Textbook questions 1, 4, and 6 of Investigation 1, "Expressions in Two Variables," p. 16

1. a. $x+y$ **b.** $6x+2y$ **c.** $5y-x$

4. a. $x+y$ **b.** $10x$ **c.** $5y$ **d.** $10x+5y$

6. a. $x+y$ **b.** $0.07x$ **c.** $0.06y$ **d.** $0.07x+0.06y$

b. Textbook questions 5, 6, and 8 of Investigation 2, "Equations in Two Variables," p. 16

5. Let ℓ be the length of the basketball court, and let w be the width.

$\therefore\ \ell+w=40$

6. Let c be the total number of judges appointed to the Supreme Court of Canada, and let q be the number of judges appointed from Quebec.

$\therefore\ c-q=6$

8. Let b be the number of bicycles, and let t be the number of tricycles.

$$\begin{aligned} \therefore\ \text{Total number of wheels} &= 2b+3t \\ 61 &= 2b+3t \end{aligned}$$

5. Textbook questions 2, 4, 7, and 8 of Investigation 3, "Systems of Equations," p. 17

2. Look for a pattern.

In the table on the left, the value of y is always 2 times the value of x. In the table on the right, the value of y is always 4 less than the value of x.

Therefore, the system of equations is as follows:

$y = 2x$ ①
$y = x - 4$ ②

4. Let p be the number of paintings produced by Picasso, and let r be the number of paintings produced by Renoir.

Therefore, the system of equations is as follows:

$p + r = 295$ ①
$p - r = 11$ ②

7. Let q be the number of quarters, and let ℓ be the number of loonies.

Therefore, the system of equations is as follows:

$q + \ell = 73$ ①
$0.25q + 1.00\ell = 37$ ②

8. Let x be one angle, and let y be the other angle.

Therefore, the system of equations is as follows:

$x + y = 180$ ①
$y = 3x - 4$ ②

6. Textbook questions 2, 4, 6, 8, and 10 of "Mental Math: Evaluating Expressions," p. 3

2.
$$\begin{aligned} 3x + 4y &= 3(3) + 4(-2) \\ &= 9 - 8 \\ &= 1 \end{aligned}$$

Section 1: Activity 1 (continued)

4. $$\begin{aligned} 2x-3y &= 2(3)-3(-2) \\ &= 6+6 \\ &= 12 \end{aligned}$$

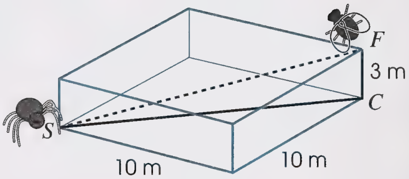
6. $$\begin{aligned} 3x-2y-5 &= 3(3)-2(-2)-5 \\ &= 9+4-5 \\ &= 8 \end{aligned}$$

8. $$\begin{aligned} 2x-y-z &= 2(2)-4-(-3) \\ &= 4-4+3 \\ &= 3 \end{aligned}$$

10. $$\begin{aligned} y+z-x &= 4+(-3)-2 \\ &= 4-3-2 \\ &= -1 \end{aligned}$$

7. Textbook questions 6, 8, and 12 in "Applications and Problem Solving," p. 15

6. Assuming that the spider doesn't already have a web from *S* to *F*, the spider would have to cross the floor from *S* to *C* and then climb the corner from *C* to *F*.



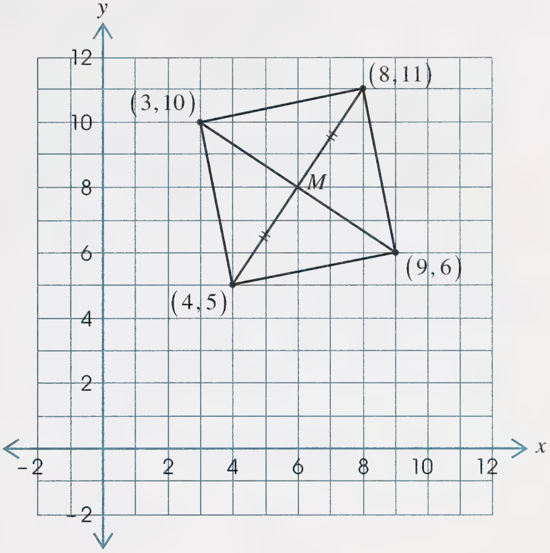
Using the Pythagorean Theorem,

$$\begin{aligned} SC &= \sqrt{10^2+10^2} \\ &= \sqrt{200} \\ &\doteq 14.1 \end{aligned}$$

$$\begin{aligned} \therefore\ SC+CF &\doteq 14.1+3 \\ &\doteq 17.1 \end{aligned}$$

The shortest distance the spider can travel to reach the fly is approximately 17.1 m.

8. First, find the midpoint, M, of the diagonal joining $(4,5)$ and $(8,11)$. Let $(x_1, y_1)=(4,5)$ and $(x_2, y_2)=(8,11)$.



$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{4+8}{2}, \frac{5+11}{2}\right)$$
$$=\left(\frac{12}{2}, \frac{16}{2}\right)$$
$$=(6,8)$$

The second diagonal will intersect the first diagonal at right angles at $M(6,8)$.

Next, find the slope, m_1, of the first diagonal, using $M(6,8)$ and $(8,11)$. Let $(x_1, y_1)=(6,8)$ and $(x_2, y_2)=(8,11)$.

$$m_1=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{11-8}{8-6}$$
$$=\frac{3}{2}$$

Therefore, the slope, m_2, of the second diagonal will be $\frac{-2}{3}$, since $m_1 \times m_2 = \frac{3}{2} \times \frac{-2}{3} = -1$.

To determine one endpoint of the second diagonal, start at $M(6,8)$ and move 3 units to the right and 2 units down.

This point is $(9,6)$.

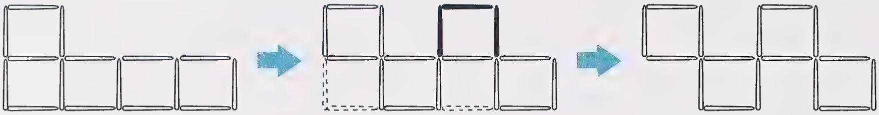
Similarly, to determine the second endpoint, start at point M and move 3 units to the left and 2 units up.

This point is $(3,10)$.

Therefore, the coordinates of the endpoints of the other diagonal are $(9,6)$ and $(3,10)$.

Section 1: Activity 1 (continued)

12.



Section 1: Activity 2

1. **Textbook questions 1.c., 1.d., 1.e., 2.b., and 2.c. of Investigation 1, "Ordered Pairs and One Equation," p. 4**

1. c. For $(-2,-4)$,

LS	RS
$2x+5y$	-24
$=2(-2)+5(-4)$	
$=-4-20$	
$=-24$	

LS $=$ RS

For $(-3,6)$,

LS	RS
$2x+5y$	-24
$=2(-3)+5(6)$	
$=-6+30$	
$=24$	

LS $\neq$ RS

For $(-12,0)$,

LS	RS
$2x+5y$	-24
$=2(-12)+5(0)$	
$=-24$	

LS $=$ RS

Therefore, $(-2,-4)$ and $(-12,0)$ satisfy $2x+5y=-24$.

d. For $(2,3)$,

LS	RS
$x-y$	-1
$=2-3$	
$=-1$	

LS $=$ RS

For $(-1,-2)$,

LS	RS
$x-y$	-1
$=-1-(-2)$	
$=1$	

LS $\neq$ RS .

For $(-5,6)$,

LS	RS
$x-y$	-1
$=-5-6$	
$=-11$	

LS $\neq$ RS

Therefore, $(2,3)$ satisfies $x-y=-1$.

e. For $(5,10)$,

LS	RS
y	$3x-4$
$=10$	$=3(5)-4$
	$=15-4$
	$=11$

LS $\neq$ RS

For $(-2,-10)$,

LS	RS
y	$3x-4$
$=-10$	$=3(-2)-4$
	$=-6-4$
	$=-10$

LS $=$ RS

For $(-3,-12)$,

LS	RS
y	$3x-4$
$=-12$	$=3(-3)-4$
	$=-9-4$
	$=-13$

LS $\neq$ RS

Therefore, $(-2,-10)$ satisfies $y=3x-4$.

Section 1: Activity 2 (continued)

2. b. If $x=7$, then

$$\begin{aligned} x-y&=5\\ 7-y&=5\\ -y&=-2\\ y&=2 \end{aligned}$$

If $x=-4$, then

$$\begin{aligned} x-y&=5\\ -4-y&=5\\ -y&=9\\ y&=-9 \end{aligned}$$

If $y=6$, then

$$\begin{aligned} x-y&=5\\ x-6&=5\\ x&=11 \end{aligned}$$

If $y=-7$, then

$$\begin{aligned} x-y&=5\\ x-(-7)&=5\\ x+7&=5\\ x&=-2 \end{aligned}$$

$\therefore\ (7,\mathbf{2}),\ (-4,\mathbf{-9}),\ (\mathbf{11},6),\ (\mathbf{-2},7)$

c. If $x=5$, then

$$\begin{aligned} 2x+y&=9\\ 2(5)+y&=9\\ 10+y&=9\\ y&=-1 \end{aligned}$$

If $y=-1$, then

$$\begin{aligned} 2x+y&=9\\ 2x+(-1)&=9\\ 2x-1&=9\\ 2x&=10\\ x&=5 \end{aligned}$$

If $x=-2$, then

$$\begin{aligned} 2x+y&=9\\ 2(-2)+y&=9\\ -4+y&=9\\ y&=13 \end{aligned}$$

If $y=-11$, then

$$\begin{aligned} 2x+y&=9\\ 2x+(-11)&=9\\ 2x-11&=9\\ 2x&=20\\ x&=10 \end{aligned}$$

$\therefore\ (5,\mathbf{-1}),\ (\mathbf{5},-1),\ (-2,\mathbf{13}),\ (\mathbf{10},-11)$

2. a. **Textbook questions 1.b., 1.d., 2.b., and 2.d. of Investigation 2, "Ordered Pairs and Two Equations," p. 5**

1. b. For $(-4, 3)$,

LS	RS
$2x+y$	-5
$=2(-4)+3$	
$=-8+3$	
$=-5$	

LS $=$ RS

LS	RS
$x+5y$	2
$=-4+5(3)$	
$=-4+15$	
$=11$	

LS $\neq$ RS

For $(-8, 11)$,

LS	RS
$2x+y$	-5
$=2(-8)+11$	
$=-16+11$	
$=-5$	

LS $=$ RS

LS	RS
$x+5y$	2
$=-8+5(11)$	
$=-8+55$	
$=47$	

LS $\neq$ RS

For $(-3, 1)$,

LS	RS
$2x+y$	-5
$=2(-3)+1$	
$=-6+1$	
$=-5$	

LS $=$ RS

LS	RS
$x+5y$	2
$=-3+5(1)$	
$=-3+5$	
$=2$	

LS $=$ RS

Therefore, $(-3, 1)$ satisfies both equations.

Section 1: Activity 2 (continued)

d. For $(3,-4)$,

LS	RS
$x+y$	-2
$=3+(-4)$	
$=-1$	

LS $\neq$ RS

LS	RS
$3x+2y$	2
$=3(3)+2(-4)$	
$=9-8$	
$=1$	

LS $\neq$ RS

For $(6,-8)$,

LS	RS
$x+y$	-2
$=6+(-8)$	
$=-2$	

LS $=$ RS

LS	RS
$3x+2y$	2
$=3(6)+2(-8)$	
$=18-16$	
$=2$	

LS $=$ RS

For $(-7,5)$,

LS	RS
$x+y$	-2
$=-7+5$	
$=-2$	

LS $=$ RS

LS	RS
$3x+2y$	2
$=3(-7)+2(5)$	
$=-21+10$	
$=-11$	

LS $\neq$ RS

Therefore, $(6,-8)$ satisfies both equations.

2. **b.** If $y = 3$, then

$$\begin{aligned} x + 2y &= 12 \\ x + 2(3) &= 12 \\ x + 6 &= 12 \\ x &= 6 \end{aligned}$$

For $(6, 3)$,

LS	RS
$2x - y$	-1
$= 2(6) - 3$	
$= 12 - 3$	
$= 9$	

LS $\neq$ RS

Therefore, $(\mathbf{6}, 3)$ satisfies the second equation but not the first.

d. Answers may vary. A sample answer is provided.

For $(-3, 1)$,

LS	RS
y	$4x - 3$
$= 1$	$= 4(-3) - 3$
	$= -12 - 3$
	$= -15$

LS $\neq$ RS

LS	RS
y	$-2x + 5$
$= 1$	$= -2(-3) + 5$
	$= 6 + 5$
	$= 11$

LS $\neq$ RS

Therefore, $(\mathbf{-3}, \mathbf{1})$ satisfies neither equation.

b. Textbook questions 2 to 4 of Investigation 3, "Problem Solving," p. 5

2. **a.** If $\ell = 16$, then

$$\begin{aligned} w - \ell &= 24 \\ w - 16 &= 24 \\ w &= 40 \end{aligned}$$

Check

For $(40, 16)$,

LS	RS
$2w - 5\ell$	0
$= 2(40) - 5(16)$	
$= 80 - 80$	
$= 0$	

LS $=$ RS

Therefore, the ordered pair is $(\mathbf{40}, 16)$.

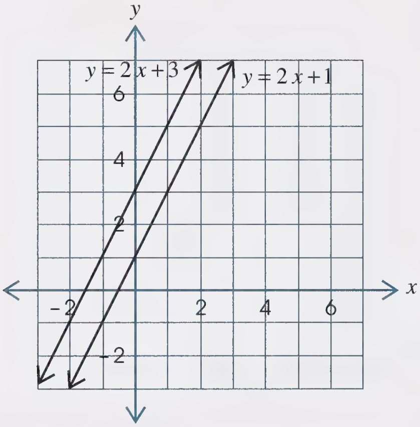
Section 1: Activity 2 (continued)

b. The average mass of an Arctic wolf is 40 kg, and the average mass of a Canadian lynx is 16 kg.

3. **a.** The graphs of both equations are the same. If you multiply the first equation by 2, you will get the second equation.

b. Answers may vary. Two more ordered pairs that satisfy both equations are $(-1, 2)$ and $(3, -2)$.

4. When you graph both equations, you obtain two parallel lines. Because parallel lines do not intersect, there are no ordered pairs that satisfy both equations.

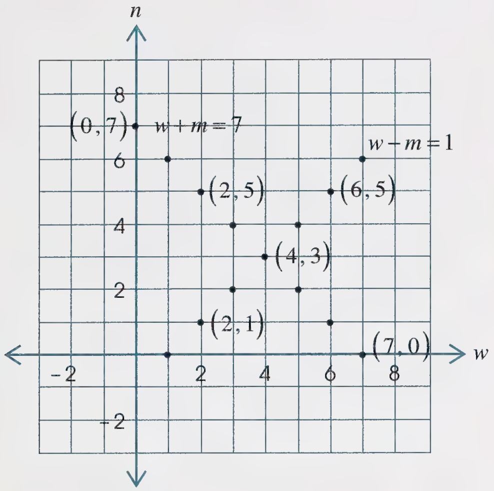


3. **a.** **Textbook questions a., b., and c. of "Explore: Use a Graph," p. 6**

a. Answers may vary. A sample answer is given.

Three ordered pairs that satisfy $w + m = 7$ are $(0, 7)$, $(2, 5)$, and $(7, 0)$.

b.



Note: You cannot join the points or have any negative coordinates. The graph consists of only whole-number values because you cannot have a fraction of a gold medal and you cannot have a negative number of gold medals.

c. Answers may vary. A sample answer is given.

Three ordered pairs that satisfy $w - m = 1$ are $(6, 5)$, $(4, 3)$, and $(2, 1)$. Refer to the answer to question b. to see the graph of $w - m = 1$.

b. Textbook questions 1, 2, 3, 5.a., and 5.b. of "Inquire," p. 6

1. a. The ordered pairs that satisfy $w + m = 7$ are $(0, 7)$, $(1, 6)$, $(2, 5)$, $(3, 4)$, $(4, 3)$, $(5, 2)$, $(6, 1)$, and $(7, 0)$. There are eight ordered pairs altogether.

b. The ordered pairs that satisfy $w - m = 1$ are $(1, 0)$, $(2, 1)$, $(3, 2)$, $(4, 3)$, $(5, 4)$, $(6, 5)$, and $(7, 6)$. There are seven ordered pairs altogether.

2. a. The ordered pair $(4, 3)$ appears to satisfy both equations.

b. You can check by substituting the ordered pair $(4, 3)$ into both equations.

Section 1: Activity 2 (continued)

c.

LS	RS
$w+m$	7
$=4+3$	
$=7$	

LS $=$ RS

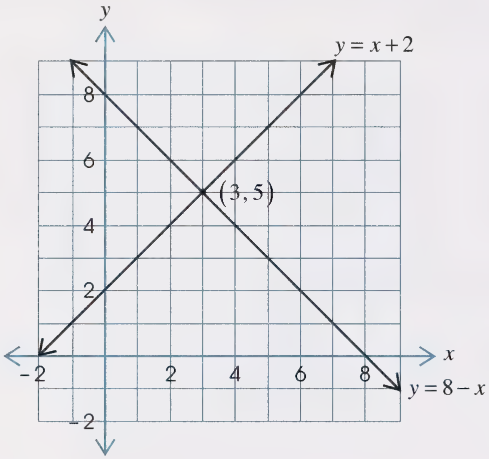
LS	RS
$w-m$	1
$=4-3$	
$=1$	

LS $=$ RS

3. **a.** Women won 4 gold medals for Canada in Barcelona.

b. Men won 3 gold medals for Canada in Barcelona.

5. **a.**



The graphs appear to intersect at $(3, 5)$.

Check

LS	RS
y	$x+2$
$=5$	$=3+2$
	$=5$

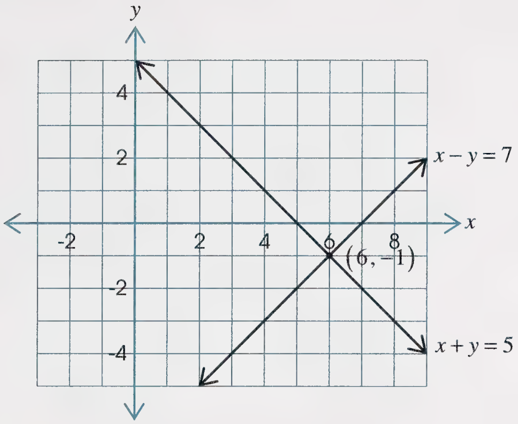
LS $=$ RS

LS	RS
y	$8-x$
$=5$	$=8-3$
	$=5$

LS $=$ RS

The coordinates of the point of intersection are $(3, 5)$.

b.



The graphs appear to intersect at $(6, -1)$.

Check

LS	RS
$x+y$	5
$=6+(-1)$	
$=5$	

LS = RS

LS	RS
$x-y$	7
$=6-(-1)$	
$=7$	

LS = RS

The coordinates of the point of intersection are $(6, -1)$.

4. a. Textbook questions 2, 4, 7, 8, 14, 16, 22, and 24 of "Practice," p. 11

2. Graph each equation using its intercepts.

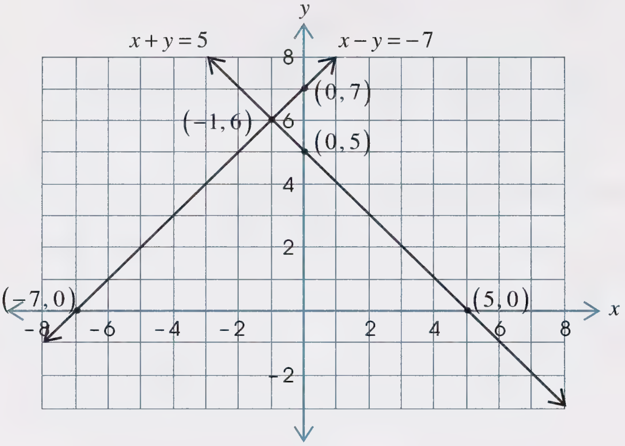
If $x = 0$, then

$$\begin{aligned} x+y&=5 &\quad \text{and} \quad x-y&=-7 \\ 0+y&=5 & 0-y&=-7 \\ y&=5 & y&=7 \end{aligned}$$

Section 1: Activity 2 (continued)

If $y = 0$, then

$$\begin{aligned} x + y &= 5 \\ x + 0 &= 5 \\ x &= 5 \end{aligned} \qquad \text{and} \qquad \begin{aligned} x - y &= -7 \\ x - 0 &= -7 \\ x &= -7 \end{aligned}$$



The graphs appear to intersect at $(-1, 6)$.

Check

LS	RS
$x + y$	5
$= -1 + 6$	
$= 5$	

LS = RS

LS	RS
$x - y$	-7
$= -1 - 6$	
$= -7$	

LS = RS

The solution is $(-1, 6)$.

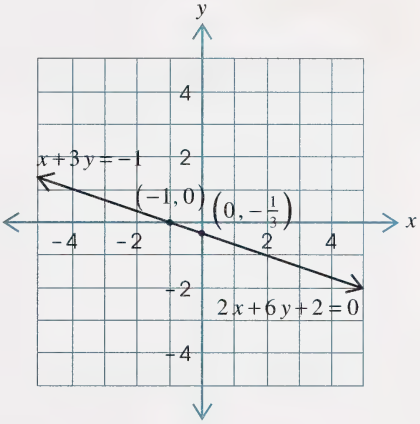
4. Graph each equation using its intercepts.

If $x=0$, then

$$\begin{aligned} x+3y&=-1 \\ 0+3y&=-1 \\ 3y&=-1 \\ y&=-\frac{1}{3} \end{aligned} \qquad \text{and} \qquad \begin{aligned} 2x+6y+2&=0 \\ 2(0)+6y&=-2 \\ 6y&=-2 \\ y&=-\frac{2}{6} \\ &=-\frac{1}{3} \end{aligned}$$

If $y=0$, then

$$\begin{aligned} x+3y&=-1 \\ x+3(0)&=-1 \\ x&=-1 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 2x+6y+2&=0 \\ 2x+6(0)&=-2 \\ 2x&=-2 \\ x&=-1 \end{aligned}$$



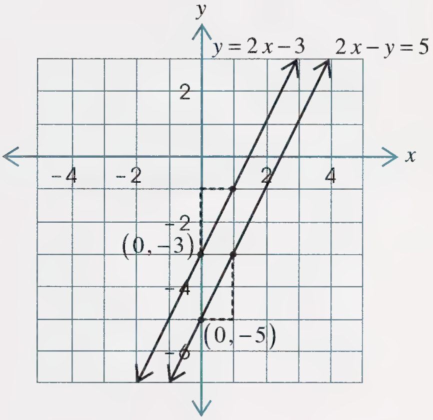
Because the graphs are the same, any ordered pair that satisfies one equation will satisfy the other. The solution is all the points on the line.

7. Write each equation in slope and y-intercept form.

$$y=2x-3 \qquad\qquad \begin{aligned} 2x-y&=5 \\ y&=2x-5 \end{aligned}$$

For $y=2x-3$, the slope is 2 and the y-intercept is -3. For $y=2x-5$, the slope is 2 and the y-intercept is -5.

Because the graphs are parallel, there is no solution.

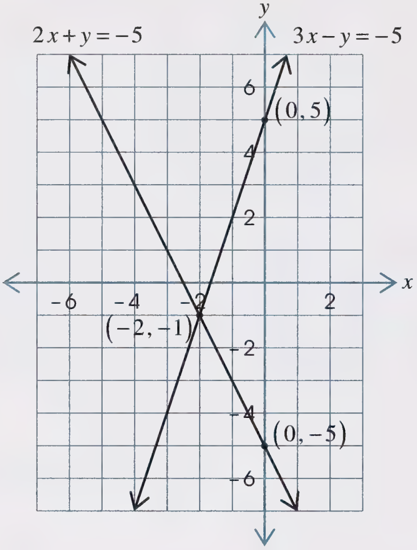


Section 1: Activity 2 (continued)

8. Write each equation in slope and *y*-intercept form.

$$2x+y=-5 \qquad\qquad 3x-y=-5$$
$$y=-2x-5 \qquad\qquad y=3x+5$$

For $y=-2x-5$, the slope is -2 and the *y*-intercept is -5. For $y=3x+5$, the slope is 3 and the *y*-intercept is 5.



The graphs appear to intersect at $(-2,-1)$.

Check

LS	RS
$2x+y$	-5
$=2(-2)+(-1)$	
$=-4-1$	
$=-5$	

LS = RS

LS	RS
$3x-y$	-5
$=3(-2)-(-1)$	
$=-6+1$	
$=-5$	

LS = RS

The solution is $(-2,-1)$.

14. Write each equation in slope and y-intercept form.

$$y=\frac{1}{2}x+3$$

$$\begin{aligned} x&=2y-6\\ 2y&=x+6\\ y&=\frac{1}{2}x+3 \end{aligned}$$

Because the graphs are the same, any ordered pair that satisfies one equation will satisfy the other.

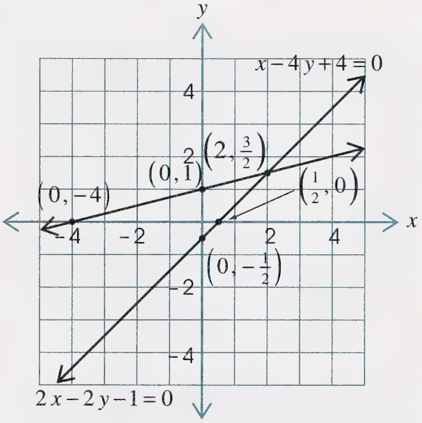
16. Graph each equation using its intercepts.

If $x=0$, then

$$\begin{aligned} 2x-2y-1&=0\\ 2(0)-2y&=1\\ -2y&=1\\ y&=-\frac{1}{2} \end{aligned} \qquad \text{and} \qquad \begin{aligned} x-4y+4&=0\\ 0-4y&=-4\\ -4y&=-4\\ y&=1 \end{aligned}$$

If $y=0$, then

$$\begin{aligned} 2x-2y-1&=0\\ 2x-2(0)&=1\\ 2x&=1\\ x&=\frac{1}{2} \end{aligned} \qquad \text{and} \qquad \begin{aligned} x-4y+4&=0\\ x-4(0)&=-4\\ x&=-4 \end{aligned}$$



It appears that the graphs intersect at $\left(2,\frac{3}{2}\right)$.

Check

LS	RS
$2x-2y-1$	0
$=2(2)-2\left(\frac{3}{2}\right)-1$	
$=4-3-1$	
$=0$	

LS = RS

LS	RS
$x-4y+4$	0
$=2-4\left(\frac{3}{2}\right)+4$	
$=2-6+4$	
$=0$	

LS = RS

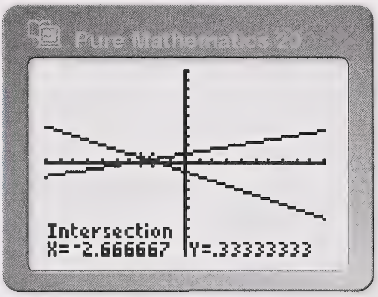
The solution is $\left(2,\frac{3}{2}\right)$.

Section 1: Activity 2 (continued)

22. $y=-0.5x-1$

$y=0.25x+1$

The solution is approximately $(-2.7, 0.3)$.

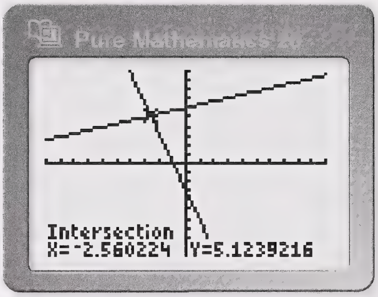


```
Intersection
X=⁻2.666667  Y=.33333333
```

24. $y=0.35x+6.02$

$y=-3.22x-3.12$

The solution is approximately $(-2.6, 5.1)$.



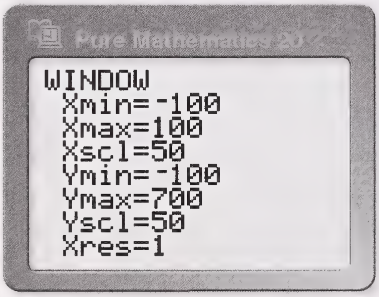
```
Intersection
X=⁻2.560224  Y=5.1239216
```

b. **Textbook questions 32, 34, 38, 40, and 42.b. of "Applications and Problem Solving," pp. 11 and 12**

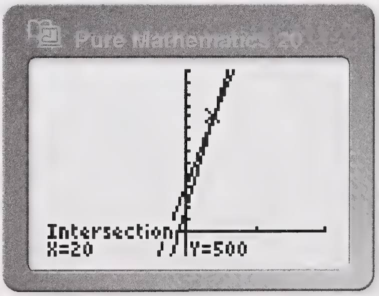
32. **a.** Graph the equations to find the point of intersection of the two lines.

$C=200+15m$

$C=100+20m$



```
WINDOW
 Xmin=⁻100
 Xmax=100
 Xscl=50
 Ymin=⁻100
 Ymax=700
 Yscl=50
 Xres=1
```



```
Intersection
X=20         Y=500
```

According to the display, the graphs intersect at $(20, 500)$.

Check

LS	RS
C	$200+15m$
$=500$	$=200+15(20)$
	$=200+300$
	$=500$

LS = RS

LS	RS
C	$100+20m$
$=500$	$=100+20(20)$
	$=100+400$
	$=500$

LS = RS

The point of intersection is $(20, 500)$.

b. After 20 months the cost of the two clubs is the same.

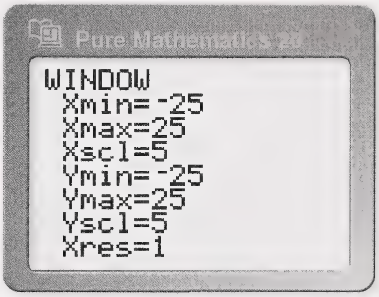
c. During the first year the Champion Health Club is less expensive. The graph of $C=100+20m$ lies below the graph of $C=200+15m$ until the twentieth month.

34. Graph the equations to find the point of intersection of the two lines. Graph v along the horizontal axis and m along the vertical axis.

$$m-v=5 \qquad 2m=3v$$

$$m=v+5 \qquad m=\frac{3}{2}v$$

According to the display, the lines intersect at $(10, 15)$.



```
WINDOW
 Xmin=-25
 Xmax=25
 Xscl=5
 Ymin=-25
 Ymax=25
 Yscl=5
 Xres=1
```

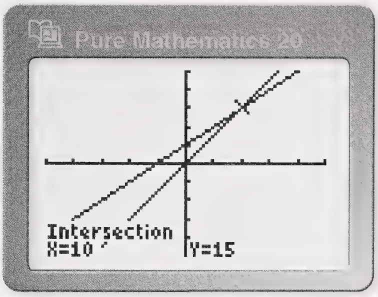
Check

LS	RS
$m-v$	5
$=15-10$	
$=5$	

LS = RS

LS	RS
$2m$	$3v$
$=2(15)$	$=3(10)$
$=30$	$=30$

LS = RS



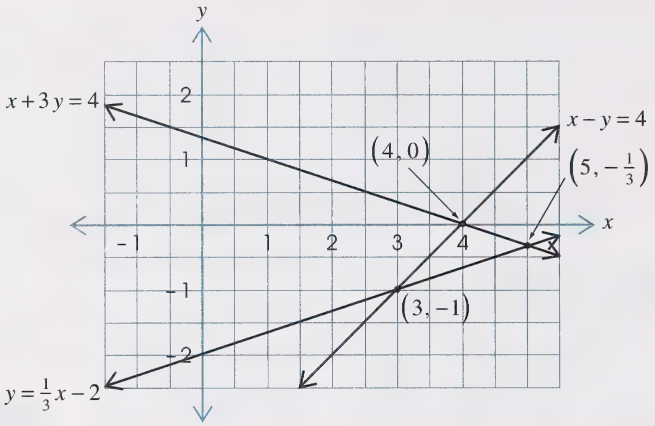
```
Intersection
X=10          Y=15
```

The point of intersection is $(10, 15)$.

Therefore, the average wind speeds for Montreal and Victoria are 15 km/h and 10 km/h, respectively.

Section 1: Activity 2 (continued)

38. Graph the equations to find the point of intersection of the three lines.



The lines $y=\frac{1}{3}x-2$ and $x-y=4$ appear to intersect at $(3,-1)$.

Check

LS	RS
y	$\frac{1}{3}x-2$
$=-1$	$=\frac{1}{\cancel{3}}(\cancel{3})-2$
	$=-1$

LS = RS

LS	RS
$x-y$	4
$=3-(-1)$	
$=3+1$	
$=4$	

LS = RS

The line $y=\frac{1}{3}x-2$ intersects $x-y=4$ at $(3,-1)$.

The lines $y=\frac{1}{3}x-2$ and $x+3y=4$ appear to intersect at $\left(5,-\frac{1}{3}\right)$.

Check

LS	RS
y	$\frac{1}{3}x-2$
$=-\frac{1}{3}$	$=\frac{1}{3}(5)-2$
	$=\frac{5}{3}-\frac{6}{3}$
	$=-\frac{1}{3}$

LS = RS

LS	RS
$x+3y$	4
$=5+3\left(-\frac{1}{3}\right)$	
$=5-1$	
$=4$	

LS = RS

The line $y=\frac{1}{3}x-2$ intersects $x+3y=4$ at $\left(5,-\frac{1}{3}\right)$.

The lines $x-y=4$ and $x+3y=4$ appear to intersect at $(4,0)$.

Check

LS	RS
$x-y$	4
$=4-0$	
$=4$	

LS = RS

LS	RS
$x+3y$	4
$=4+3(0)$	
$=4$	

LS = RS

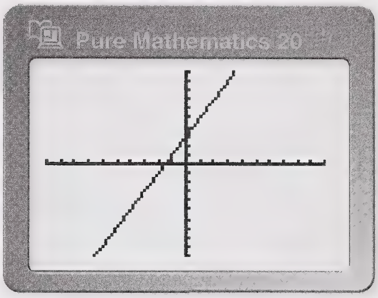
The line $x-y=4$ intersects $x+3y=4$ at $(4,0)$.

Therefore, the coordinates of the triangle are $(3,-1)$, $\left(5,-\frac{1}{3}\right)$, and $(4,0)$.

Section 1: Activity 2 (continued)

40. Answers may vary. A sample answer is given.

a. Graph $y = 2x + 3$.



Points that satisfy the inequality $y < 2x + 3$ lie below the line $y = 2x + 3$. Three such points are $(0, 0)$, $(-1, -1)$, and $(2, -4)$.

Check

For $(0, 0)$,

LS	RS
y	$2x + 3$
$= 0$	$= 2(0) + 3$
	$= 3$

LS $<$ RS

For $(-1, -1)$,

LS	RS
y	$2x + 3$
$= -1$	$= 2(-1) + 3$
	$= 1$

LS $<$ RS

For $(2, -4)$,

LS	RS
y	$2x + 3$
$= -4$	$= 2(2) + 3$
	$= 4 + 3$
	$= 7$

LS $<$ RS

Three points that satisfy $y < 2x + 3$ are $(0, 0)$, $(-1, -1)$, and $(2, -4)$.

b. Refer to the graph in the answer to question 40.a. Points that satisfy the inequality $y > 2x+3$ lie above the line $y = 2x+3$. Three such points are $(0,5)$, $(-3,3)$, and $(1,10)$.

Check

For $(0,5)$,

LS	RS
y	$2x+3$
$=5$	$=2(0)+3$
	$=3$

LS $>$ RS

For $(-3,3)$,

LS	RS
y	$2x+3$
$=3$	$=2(-3)+3$
	$=-6+3$
	$=-3$

LS $>$ RS

For $(1,10)$,

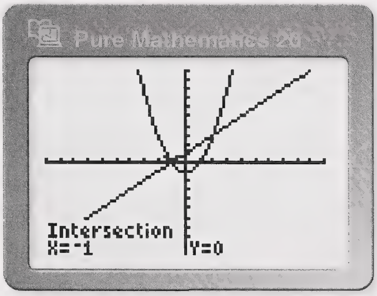
LS	RS
y	$2x+3$
$=10$	$=2(1)+3$
	$=5$

LS $>$ RS

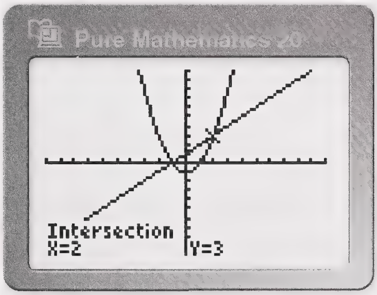
Three points that satisfy the inequality $y > 2x+3$ are $(0,5)$, $(-3,3)$, and $(1,10)$.

42. b. $y = x^2 - 1$ ①

$y = x + 1$ ②



Intersection
X=-1 Y=0



Intersection
X=2 Y=3

The solution of the system are the points of intersection. The calculator indicates that the curves cross at $(-1,0)$ and $(2,3)$.

Section 1: Activity 2 (continued)

Check

For $(-1, 0)$,

LS	RS
y	$x^2 - 1$
$= 0$	$= (-1)^2 - 1$
	$= 1 - 1$
	$= 0$

LS = RS

LS	RS
y	$x + 1$
$= 0$	$= (-1) + 1$
	$= 0$

LS = RS

For $(2, 3)$,

LS	RS
y	$x^2 - 1$
$= 3$	$= 2^2 - 1$
	$= 4 - 1$
	$= 3$

LS = RS

LS	RS
y	$x + 1$
$= 3$	$= 2 + 1$
	$= 3$

LS = RS

The solution is $(-1, 0)$ and $(2, 3)$.

5. a. Textbook questions 26, 28, and 29 of "Practice," p. 11

26. Write each equation in slope and y-intercept form.

$$\begin{aligned} 3x - y &= 0 \\ -y &= -3x \\ y &= 3x + 0 \end{aligned}$$

The slope is 3, and the y-intercept is 0.

$$\begin{aligned} 6x-2y&=3 \\ -2y&=-6x+3 \\ y&=3x-\frac{3}{2} \end{aligned}$$

The slope is 3, and the y-intercept is $-\frac{3}{2}$.

Because the slopes of the lines are equal and the y-intercepts are different, the lines are parallel and do not intersect. Therefore, the system has no solution.

28. Write each equation in slope and y-intercept form.

$$\begin{aligned} x+4y&=8 \\ 4y&=-x+8 \\ y&=-\frac{1}{4}x+2 \end{aligned}$$

The slope is $-\frac{1}{4}$, and the y-intercept is 2.

$$\begin{aligned} y+2x&=0 \\ y&=-2x+0 \end{aligned}$$

The slope is -2, and the y-intercept is 0.

Because the slopes of the lines differ, the lines cross at exactly one point. Therefore, the system has one solution.

29. Write each equation in slope and y-intercept form.

$$\begin{aligned} 2y&=3x-1 \\ y&=\frac{3}{2}x-\frac{1}{2} \end{aligned}$$

The slope is $\frac{3}{2}$ and the y-intercept is $-\frac{1}{2}$.

$$\begin{aligned} 8y-4&=12x \\ 8y&=12x+4 \\ y&=\frac{3}{2}x+\frac{1}{2} \end{aligned}$$

The slope is $\frac{3}{2}$ and the y-intercept is $\frac{1}{2}$.

Because the slopes of the lines are equal and the y-intercepts are different, the lines are parallel and do not intersect. Therefore, the system has no solution.

Section 1: Activity 2 (continued)

b. Textbook questions 44 and 46 of "Applications and Problem Solving," p. 12

44. Answers may vary. Sample answers are given.

a. The following system of equations has the point $(3, 2)$ as its only solution:

$$y = 3x - 7$$
$$y = -x + 5$$

Because the equations have different slopes, the lines intersect at exactly one point.

Check

For $(3, 2)$,

LS	RS
y	$3x - 7$
$= 2$	$= 3(3) - 7$
	$= 9 - 7$
	$= 2$

LS = RS

LS	RS
y	$-x + 5$
$= 2$	$= -3 + 5$
	$= 2$

LS = RS

The solution is $(3, 2)$.

b. From the answer to question 44.a., you know the graph of $y = 3x - 7$ passes through $(3, 2)$. So, to obtain a second equation that has $(3, 2)$ as one of an infinite number of solutions, multiply $y = 3x - 7$ by any non-zero constant.

Try multiplying by 2.

$$2(y = 3x - 7)$$
$$2y = 6x - 14$$

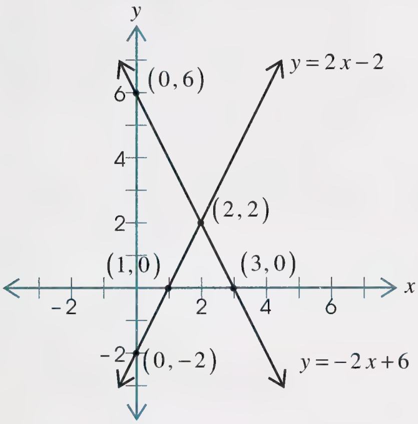
Therefore, the following system has $(3, 2)$ as one of an infinite number of solutions.

$y = 3x - 7$ ①

$2y = 6x - 14$ ②

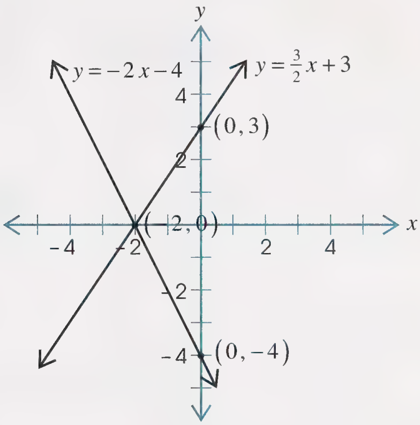
46. Answers may vary. Sample answers are given.

a. $y=2x-2$ ①
$y=-2x+6$ ②



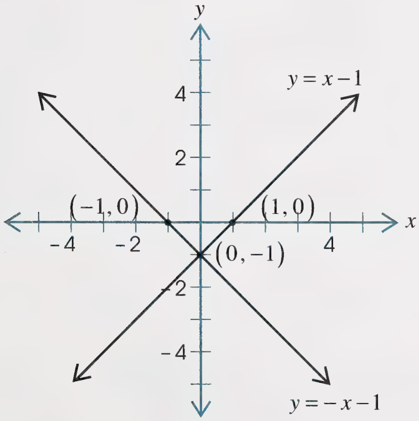
Both graphs have different x- and y-intercepts.

b. $y=\frac{3}{2}x+3$ ①
$y=-2x-4$ ②



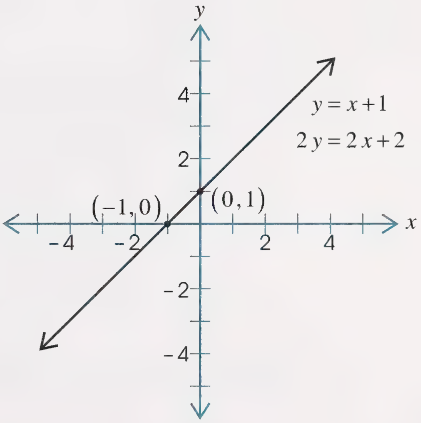
Both graphs have the same x-intercept and different y-intercepts.

c. $y=x-1$ ①
$y=-x-1$ ②



Both graphs have different x-intercepts and the same y-intercept.

d. $y=x+1$ ①
$2y=2x+2$ ②

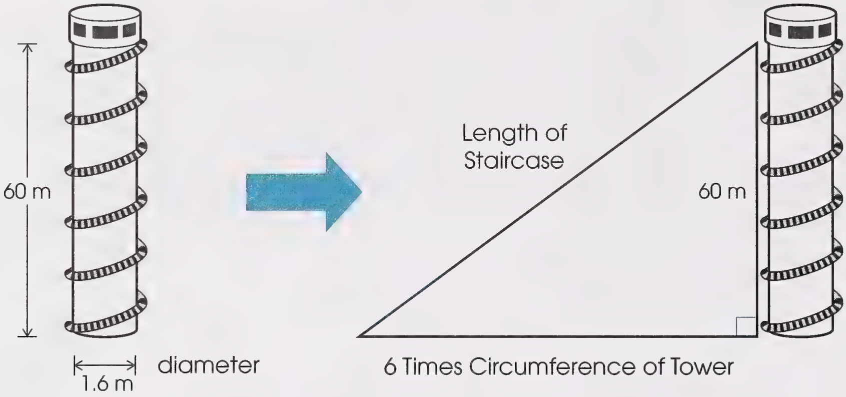


Both graphs have the same x- and y-intercepts.

Section 1: Activity 2 (continued)

6. Textbook questions 1, 2, 3, 5, and 11 of "Applications and Problem Solving," p. 15

1. Unwind the staircase from the tower to form a right triangle.



Let L be the length of the staircase, and let B be the base of the right triangle.

$$\begin{aligned} \therefore\ B &= 6\times\text{Circumference of the Tower} \\ &= 6\times(2\pi r) \\ &= 12\pi r \\ &= 12\pi(0.8) \\ &= 9.6\pi \end{aligned}$$

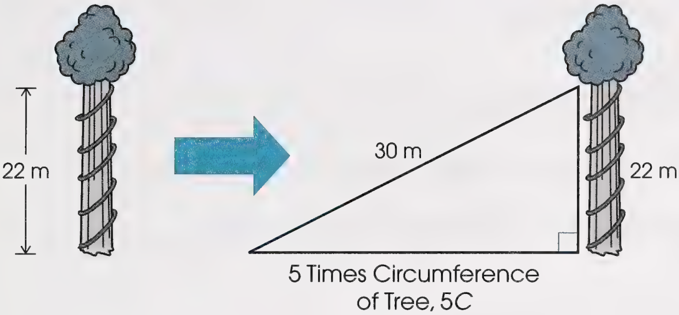
diameter = 1.6 m

Use the Pythagorean Theorem to find L.

$$\begin{aligned} L^2 &= B^2+60^2 \\ L^2 &= (9.6\pi)^2+60^2 \\ L &= \sqrt{(9.6\pi)^2+60^2} \\ &\doteq 67 \end{aligned}$$

The length of the stairs is approximately 67 m.

2.



Use the Pythagorean Theorem to find the circumference of the tree.

$$\begin{aligned}(5C)^2 &= 30^2 - 22^2\\ 25C^2 &= 900 - 484\\ 25C^2 &= 416\\ C^2 &= 16.64\\ C &= \sqrt{16.64}\end{aligned}$$

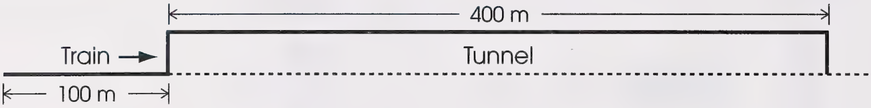
Now, find the radius of the tree.

$$\begin{aligned}C &= 2\pi r\\ r &= \frac{C}{2\pi}\\ &= \frac{\sqrt{16.64}}{2\pi}\\ &\doteq 0.65\ \text{m}\\ &\doteq 65\ \text{cm}\end{aligned}$$

The radius of the tree is approximately 65 cm.

Section 1: Activity 2 (continued)

3.



From the end of the train to the end of the tunnel is $100 + 400 = 500\ \text{m}$.

Recall that $\text{distance} = \text{speed} \times \text{time}$.

$$\begin{aligned}\therefore\ \text{time} &= \frac{\text{distance}}{\text{speed}}\\ &= \frac{0.5}{90}\\ &= \frac{5}{900}\\ &= \frac{1}{180}\end{aligned}$$

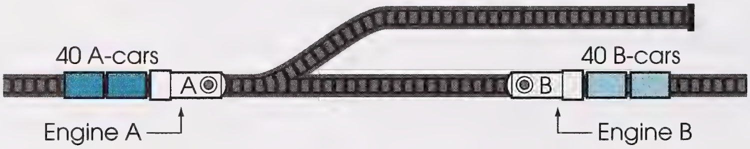
500 m = 0.5 km

Convert the time from hours to seconds.

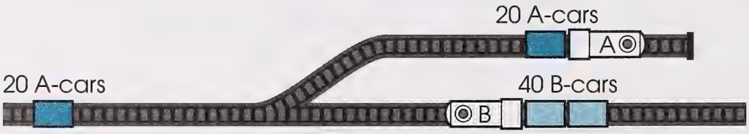
$$\begin{aligned}\frac{1}{180}\cancel{\text{h}} \times \frac{3600\ \text{s}}{1\ \cancel{\text{h}}} &= \frac{3600\ \text{s}}{180}\\ &= 20\ \text{s}\end{aligned}$$

It would take the train 20 s to pass completely through the tunnel.

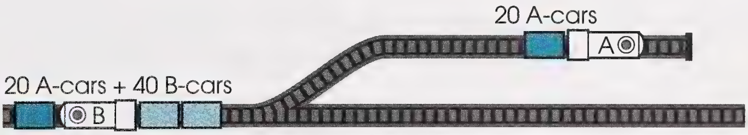
5. The following diagram shows the initial positions of the trains.



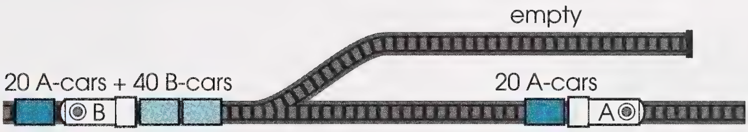
Step 1: Train A disconnects 20 cars from the rear, leaving them on the track, and pulls 20 cars onto the siding.



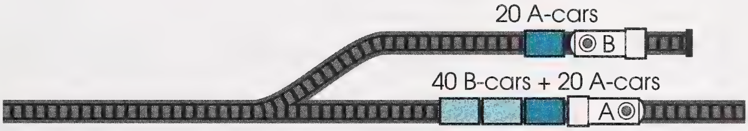
Step 2: Engine B pulls ahead and pushes the 20 A-cars and pulls 40 B-cars well clear of the siding.



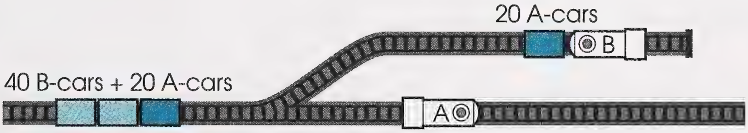
Step 3: Engine A puts its 20 A-cars out on the main track and pulls well ahead to clear the siding.



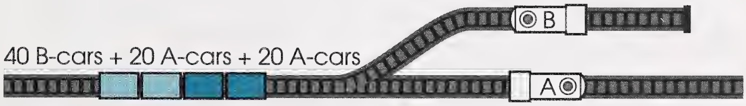
Step 4: Engine B, in between the 20 A-cars and the 40 B-cars, pushes the 40 B-cars up to the 20 A-cars connected to Engine A; then Engine B backs up and pulls the 20 A-cars onto the siding.



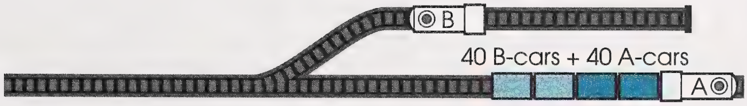
Step 5: Engine A pushes 40 B-cars and the 20 A-cars well past the siding; then Engine A disconnects and pulls well ahead on the main line.



Step 6: Engine B pushes 20 A-cars back onto the main line; then Engine B pulls back and waits on the siding. All cars are now connected.

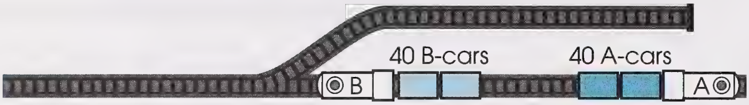


Step 7: Engine A backs up, connects to all 80 cars, and pulls far enough forward so the end car clears the siding.

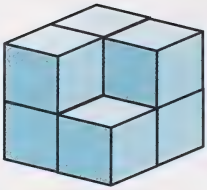


Section 1: Activity 2 (continued)

Step 8: The B-cars are separated from the A-cars and Train A goes on its way. Meanwhile, Engine B connects to its 40 B-cars and proceeds on its way.



11.



Section 1: Activity 3

1. **a.** Because $t-15\ell=0$ can be written $t=15\ell$, the equation tells you that there are 15 tigers for every leopard.

   b. The equation $t+\ell=480$ tells you there are 480 tigers and leopards altogether.

2. **Textbook question 4 of "Inquire," p. 21**

   4. An ordered pair, of the form (t,ℓ), that satisfies the system of equations is $(450,30)$.

Check

LS	RS
$t-15\ell$	0
$=450-15(30)$	
$=450-450$	
$=0$	

LS = RS

LS	RS
$t+\ell$	480
$=450+30$	
$=480$	

LS = RS

3. Textbook questions 5 and 7.c. of "Inquire," p. 21

5. a.

$$\begin{aligned} t+\ell&=480\\ t&=480-\ell \end{aligned}$$

b.

$$\begin{aligned} t-15\ell&=0\\ (480-\ell)-15\ell&=0 \quad \longleftarrow \text{Substitute } 480-\ell \text{ for } t.\\ 480-16\ell&=0\\ 480&=16\ell\\ 30&=\ell\\ \ell&=30 \end{aligned}$$

c.

$$\begin{aligned} t+\ell&=480\\ t+30&=480 \quad \longleftarrow \text{Substitute 30 for } \ell.\\ t&=450 \end{aligned}$$

d. Yes, the ordered pair $(t,\ell)=(450,30)$ is the same as in question 4.

7. c.

$$\begin{aligned} 2x+y&=2 \quad ①\\ 3x+2y&=1 \quad ② \end{aligned}$$

Solve for y in ①.

$$\begin{aligned} 2x+y&=2\\ y&=-2x+2 \end{aligned}$$

Substitute $-2x+2$ for y into ②.

$$\begin{aligned} 3x+2y&=1\\ 3x+2(-2x+2)&=1\\ 3x-4x+4&=1\\ -x+4&=1\\ -x&=-3\\ x&=3 \end{aligned}$$

Section 1: Activity 3 (continued)

Substitute $x = 3$ into ①.

$$\begin{aligned} 2x + y &= 2 \\ 2(3) + y &= 2 \\ 6 + y &= 2 \\ y &= -4 \end{aligned}$$

Check

LS	RS
$2x + y$	2
$= 2(3) + (-4)$	
$= 6 - 4$	
$= 2$	

LS = RS

LS	RS
$3x + 2y$	1
$= 3(3) + 2(-4)$	
$= 9 - 8$	
$= 1$	

LS = RS

Therefore, the solution is $(3, -4)$.

4. **Textbook questions 10, 12, 13, and 15 of "Practice," p. 26**

10. $2x - y = 13$ ①
$x + 2y = -6$ ②

Solve for y in ①.

$$\begin{aligned} 2x - y &= 13 \\ -y &= -2x + 13 \\ y &= 2x - 13 \end{aligned}$$

Substitute $2x - 13$ for y into ②.

$$\begin{aligned} x + 2y &= -6 \\ x + 2(2x - 13) &= -6 \\ x + 4x - 26 &= -6 \\ 5x - 26 &= -6 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

Substitute $x=4$ into ①.

$$\begin{aligned} 2x-y&=13 \\ 2(4)-y&=13 \\ 8-y&=13 \\ -y&=5 \\ y&=-5 \end{aligned}$$

Check

LS	RS
$2x-y$	13
$=2(4)-(-5)$	
$=8+5$	
$=13$	

LS = RS

LS	RS
$x+2y$	-6
$=4+2(-5)$	
$=4-10$	
$=-6$	

LS = RS

The solution is $(4,-5)$.

12. $2x+3y=5$ ①

$x-4y=-14$ ②

Solve for x in ②.

$$\begin{aligned} x-4y&=-14 \\ x&=4y-14 \end{aligned}$$

Substitute $4y-14$ for x into ①.

$$\begin{aligned} 2x+3y&=5 \\ 2(4y-14)+3y&=5 \\ 8y-28+3y&=5 \\ 11y-28&=5 \\ 11y&=33 \\ y&=3 \end{aligned}$$

Section 1: Activity 3 (continued)

Substitute $y = 3$ into ②.

$$\begin{aligned} x - 4y &= -14 \\ x - 4(3) &= -14 \\ x - 12 &= -14 \\ x &= -2 \end{aligned}$$

Check

LS	RS
$2x + 3y$	5
$= 2(-2) + 3(3)$	
$= -4 + 9$	
$= 5$	

LS = RS

LS	RS
$x - 4y$	-14
$= -2 - 4(3)$	
$= -2 - 12$	
$= -14$	

LS = RS

The solution is $(-2, 3)$.

13. $2c - d + 2 = 0$ ①

$3c + 2d + 10 = 0$ ②

Solve for d in ①.

$$\begin{aligned} 2c - d + 2 &= 0 \\ 2c + 2 &= d \\ d &= 2c + 2 \end{aligned}$$

Substitute $2c + 2$ for d into ②.

$$\begin{aligned} 3c + 2d + 10 &= 0 \\ 3c + 2(2c + 2) + 10 &= 0 \\ 3c + 4c + 4 + 10 &= 0 \\ 7c + 14 &= 0 \\ 7c &= -14 \\ c &= -2 \end{aligned}$$

Substitute $c = -2$ into ①.

$$\begin{aligned} 2c - d + 2 &= 0 \\ 2(-2) - d + 2 &= 0 \\ -4 - d + 2 &= 0 \\ -2 - d &= 0 \\ -d &= 2 \\ d &= -2 \end{aligned}$$

Check

LS	RS
$2c - d + 2$	0
$= 2(-2) - (-2) + 2$	
$= -4 + 2 + 2$	
$= 0$	

LS = RS

LS	RS
$3c + 2d + 10$	0
$= 3(-2) + 2(-2) + 10$	
$= -6 - 4 + 10$	
$= 0$	

LS = RS

The solution is $(-2, -2)$.

15. $a + 4b = 3$ ①

$5b = -2a + 3$ ②

Solve for a in ①.

$$\begin{aligned} a + 4b &= 3 \\ a &= -4b + 3 \end{aligned}$$

Substitute $-4b + 3$ for a into ②.

$$\begin{aligned} 5b &= -2a + 3 \\ 5b &= -2(-4b + 3) + 3 \\ 5b &= 8b - 6 + 3 \\ 5b &= 8b - 3 \\ -3b &= -3 \\ b &= 1 \end{aligned}$$

Section 1: Activity 3 (continued)

Substitute $b = 1$ into ①.

$$\begin{aligned} a + 4b &= 3 \\ a + 4(1) &= 3 \\ a + 4 &= 3 \\ a &= -1 \end{aligned}$$

Check

LS	RS
$a + 4b$	3
$= -1 + 4(1)$	
$= -1 + 4$	
$= 3$	

LS = RS

LS	RS
$5b$	$-2a + 3$
$= 5(1)$	$= -2(-1) + 3$
$= 5$	$= 2 + 3$
	$= 5$

LS = RS

The solution is $(-1, 1)$.

5. **a. Textbook questions 14, 17, 21, 22, and 24 of "Practice," p. 26**

14. $4x - y = 3$ ①

$6x - 2y = 5$ ②

Solve for y in ①.

$$\begin{aligned} 4x - y &= 3 \\ -y &= -4x + 3 \\ y &= 4x - 3 \end{aligned}$$

Substitute $4x - 3$ for y into ②.

$$\begin{aligned} 6x - 2y &= 5 \\ 6x - 2(4x - 3) &= 5 \\ 6x - 8x + 6 &= 5 \\ -2x + 6 &= 5 \\ -2x &= -1 \\ x &= \frac{1}{2} \end{aligned}$$

Substitute $x=\frac{1}{2}$ into ①.

$$\begin{aligned} 4x-y&=3\\ \overset{2}{\cancel{4}}\left(\frac{1}{\underset{1}{\cancel{2}}}\right)-y&=3\\ 2-y&=3\\ -y&=1\\ y&=-1 \end{aligned}$$

Check

	LS	RS
$4x-y$		3
$=\overset{2}{\cancel{4}}\left(\frac{1}{\underset{1}{\cancel{2}}}\right)-(-1)$		
$=2+1$		
$=3$		

LS = RS

	LS	RS
$6x-2y$		5
$=\overset{3}{\cancel{6}}\left(\frac{1}{\underset{1}{\cancel{2}}}\right)-2(-1)$		
$=3+2$		
$=5$		

LS = RS

The solution is $\left(\frac{1}{2},-1\right)$.

17. $2x-5y=12$ ①

$x+10y=-9$ ②

Solve for x in ②.

$$\begin{aligned} x+10y&=-9\\ x&=-10y-9 \end{aligned}$$

Substitute $-10y-9$ for x into ①.

$$\begin{aligned} 2x-5y&=12\\ 2(-10y-9)-5y&=12\\ -20y-18-5y&=12\\ -25y&=30\\ y&=\frac{30}{-25}\\ &=-\frac{6}{5} \end{aligned}$$

Section 1: Activity 3 (continued)

Substitute $y=-\frac{6}{5}$ into ②.

$$x+10y=-9$$
$$x+\overset{2}{\cancel{10}}\left(-\frac{6}{\underset{1}{\cancel{5}}}\right)=-9$$
$$x-12=-9$$
$$x=3$$

Check

LS	RS
$2x-5y$	12
$=2(3)-\cancel{5}\left(-\frac{6}{\cancel{5}}\right)$	
$=6+6$	
$=12$	

LS = RS

LS	RS
$x+10y$	-9
$=3+\overset{2}{\cancel{10}}\left(-\frac{6}{\underset{1}{\cancel{5}}}\right)$	
$=3-12$	
$=-9$	

LS = RS

The solution is $\left(3,-\frac{6}{5}\right)$.

21. $5=2y-x$ ①
$7=3y-2x$ ②

Solve for x in ①.

$$5=2y-x$$
$$x=2y-5$$

Substitute $2y-5$ for x in ②.

$$7=3y-2x$$
$$7=3y-2(2y-5)$$
$$7=3y-4y+10$$
$$7=-y+10$$
$$y=3$$

Substitute $y=3$ into ①.

$$\begin{aligned}5&=2y-x\\5&=2(3)-x\\5&=6-x\\x&=1\end{aligned}$$

Check

LS	RS
5	$2y-x$
	$=2(3)-1$
	$=6-1$
	$=5$

LS = RS

LS	RS
7	$3y-2x$
	$=3(3)-2(1)$
	$=9-2$
	$=7$

LS = RS

The solution is $(1,3)$.

22. $x+7y=1$ ①

$3x-14y=-7$ ②

Solve for x in ①.

$$\begin{aligned}x+7y&=1\\x&=1-7y\end{aligned}$$

Substitute $1-7y$ for x into ②.

$$\begin{aligned}3x-14y&=-7\\3(1-7y)-14y&=-7\\3-21y-14y&=-7\\3-35y&=-7\\-35y&=-10\\y&=\frac{-10}{-35}\\&=\frac{2}{7}\end{aligned}$$

Section 1: Activity 3 (continued)

Substitute $y = \frac{2}{7}$ into ①.

$$\begin{aligned} x + 7y &= 1 \\ x + \cancel{7}\left(\frac{2}{\cancel{7}}\right) &= 1 \\ x + 2 &= 1 \\ x &= -1 \end{aligned}$$

Check

LS	RS
$x + 7y$	1
$= -1 + \cancel{7}\left(\frac{2}{\cancel{7}}\right)$	
$= -1 + 2$	
$= 1$	

LS = RS

LS	RS
$3x - 14y$	-7
$= 3(-1) - \overset{2}{\cancel{14}}\left(\frac{2}{\underset{1}{\cancel{7}}}\right)$	
$= -3 - 4$	
$= -7$	

LS = RS

The solution is $\left(-1, \frac{2}{7}\right)$.

24. $3x - 2y = -12$ ①
$x - 4y = 8$ ②

Solve for x in ②.

$$\begin{aligned} x - 4y &= 8 \\ x &= 4y + 8 \end{aligned}$$

Substitute $4y + 8$ for x into ①.

$$\begin{aligned} 3x - 2y &= -12 \\ 3(4y + 8) - 2y &= -12 \\ 12y + 24 - 2y &= -12 \\ 10y &= -36 \\ y &= \frac{-36}{10} \\ &= -\frac{18}{5} \end{aligned}$$

Substitute $y=-\frac{18}{5}$ into ②.

$$\begin{aligned} x-4y&=8 \\ x-4\left(-\frac{18}{5}\right)&=8 \\ x+\frac{72}{5}&=8 \\ x&=\frac{40}{5}-\frac{72}{5} \\ &=-\frac{32}{5} \end{aligned}$$

Check

LS	RS
$3x-2y$	-12
$=3\left(-\frac{32}{5}\right)-2\left(-\frac{18}{5}\right)$	
$=-\frac{96}{5}+\frac{36}{5}$	
$=-\frac{60}{5}$	
$=-12$	

LS = RS

LS	RS
$x-4y$	8
$=\left(-\frac{32}{5}\right)-4\left(-\frac{18}{5}\right)$	
$=\frac{-32}{5}+\frac{72}{5}$	
$=\frac{40}{5}$	
$=8$	

LS = RS

The solution is $\left(-\frac{32}{5},-\frac{18}{5}\right)$.

Section 1: Activity 3 (continued)

b. **Textbook question 29.c. of "Applications and Problem Solving," p. 26**

29. **c.** $2(3x-1)-(y+4)=-7$ ①

$4(1-2x)-3(3-y)=-12$ ②

Simplify ①.

$$\begin{aligned}2(3x-1)-(y+4)&=-7\\6x-2-y-4&=-7\\6x-y-6&=-7\\6x-y&=-1 \quad ③\end{aligned}$$

Simplify ②.

$$\begin{aligned}4(1-2x)-3(3-y)&=-12\\4-8x-9+3y&=-12\\-8x+3y-5&=-12\\-8x+3y&=-7 \quad ④\end{aligned}$$

Solve for y in ③.

$$\begin{aligned}6x-y&=-1\\y&=6x+1\end{aligned}$$

Substitute $6x+1$ for y into ④.

$$\begin{aligned}-8x+3y&=-7\\-8x+3(6x+1)&=-7\\-8x+18x+3&=-7\\10x+3&=-7\\10x&=-10\\x&=-1\end{aligned}$$

Substitute for $x=-1$ into ③.

$$\begin{aligned}6x-y&=-1\\6(-1)-y&=-1\\-6-y&=-1\\y&=-5\end{aligned}$$

Check

LS	RS
$2(3x-1)-(y+4)$	-7
$=2[3(-1)-1]-(-5+4)$	
$=2(-4)-(-1)$	
$=-8+1$	
$=-7$	

LS = RS

LS	RS
$4(1-2x)-3(3-y)$	-12
$=4[1-2(-1)]-3[3-(-5)]$	
$=4(3)-3(8)$	
$=12-24$	
$=-12$	

LS = RS

The solution is $(-1,-5)$.

6. Textbook question 37 of "Applications and Problem Solving," p. 27

37. Let x be the amount invested in a term deposit, and let y be the amount invested in a treasury bill.

Organize the information in a table.

	Hakim's Investments		
	at 4%	at 5%	Total
Amount Invested ($)	x	y	15 000
Interest Earned ($)	$0.04x$	$0.05y$	690

Write the system of equations.

$x+y=15\,000$ ①

$0.04x+0.05y=690$ ②

Solve for x in ①.

$$x+y=15\,000$$
$$x=15\,000-y$$

Section 1: Activity 3 (continued)

Substitute $15\,000 - y$ for x into ②.

$$\begin{aligned} 0.04x + 0.05y &= 690 \\ 0.04(15\,000 - y) + 0.05y &= 690 \\ 600 - 0.04y + 0.05y &= 690 \\ 0.01y &= 90 \\ y &= 9000 \end{aligned}$$

Substitute $y = 9000$ into ①.

$$\begin{aligned} x + y &= 15\,000 \\ x + 9000 &= 15\,000 \\ x &= 6000 \end{aligned}$$

Check

LS	RS
$x+y$	15 000
$=6000+9000$	
$=15\,000$	

LS = RS

LS	RS
$0.04x+0.05y$	690
$=0.04(6000)+0.05(9000)$	
$=240+450$	
$=690$	

LS = RS

Therefore, Hakim invested \$6000 in a term deposit and \$9000 in a treasury bill.

7. Textbook questions 40 and 41 of "Applications and Problem Solving," p. 27

40. Let x be the volume of the weaker vinegar solution, and let y be the volume of the stronger vinegar solution.

Organize the information in a table.

	Vinegar Solutions		
	5% Acid	**10% Acid**	**9% Acid**
Volume of Solution (mL)	x	y	50
Volume of Acetic Acid (mL)	$0.05x$	$0.10y$	$0.09(50)$

Write a system of equations.

$x + y = 50$ ①

$0.05x + 0.10y = 4.5$ ②

$0.09(50) = 4.5$

Solve for x in ①.

$$\begin{aligned} x + y &= 50 \\ x &= 50 - y \end{aligned}$$

Substitute $50 - y$ for x into ②.

$$\begin{aligned} 0.05x + 0.10y &= 4.5 \\ 0.05(50 - y) + 0.10y &= 4.5 \\ 2.5 - 0.05y + 0.10y &= 4.5 \\ 2.5 + 0.05y &= 4.5 \\ 0.05y &= 2.0 \\ y &= 40 \end{aligned}$$

Substitute $y = 40$ into ①.

$$\begin{aligned} x + y &= 50 \\ x + 40 &= 50 \\ x &= 10 \end{aligned}$$

Check

LS	RS
$x + y$	50
$= 10 + 40$	
$= 50$	

LS = RS

LS	RS
$0.05x + 0.10y$	4.5
$= 0.05(10) + 0.10(40)$	
$= 0.5 + 4.0$	
$= 4.5$	

LS = RS

Therefore, 10 mL of the weaker vinegar solution and 40 mL of the stronger vinegar solution must be mixed to make 50 mL of a 9% vinegar solution.

Section 1: Activity 3 (continued)

41. Let x be the mass of the 18-carat gold, and let y be the mass of the 9-carat gold.

Organize the information in a table.

	Gold Jewellery		
	18-carat	**9-carat**	**15-carat**
Mass of Metal (g)	x	y	150
Mass of Gold (g)	$0.75x$	$0.375y$	$0.625(150)$

Write the system of equations.

$$x + y = 150 \quad ①$$

$$0.75x + 0.375y = 93.75 \quad ②$$

$0.625(150) = 93.75$

Solve for x in ①.

$$\begin{aligned} x + y &= 150 \\ x &= 150 - y \end{aligned}$$

Substitute $150 - y$ for x into ②.

$$\begin{aligned} 0.75x + 0.375y &= 93.75 \\ 0.75(150 - y) + 0.375y &= 93.75 \\ 112.5 - 0.75y + 0.375y &= 93.75 \\ 112.5 - 0.375y &= 93.75 \\ -0.375y &= -18.75 \\ y &= 50 \end{aligned}$$

Substitute $y = 50$ into ①.

$$\begin{aligned} x + y &= 150 \\ x + 50 &= 150 \\ x &= 100 \end{aligned}$$

Check

LS	RS
$x+y$	150
$=100+50$	
$=150$	

LS = RS

LS	RS
$0.75x+0.375y$	93.75
$=0.75(100)+0.375(50)$	
$=75+18.75$	
$=93.75$	

LS = RS

Therefore, the jewellery maker should melt and mix 100 g of 18-carat gold and 50 g of 9-carat gold in order to make 150 g of 15-carat gold.

8. Textbook questions 30, 32, 33, 35, 36, 42, and 46 of "Applications and Problem Solving," pp. 26 and 27

30. Let x be one number, and let y be the other number.

$\therefore\ x+y=752$ ①

$x-y=174$ ②

Solve for x in ①.

$$x+y=752$$
$$x=752-y$$

Substitute $752-y$ for x into ②.

$$x-y=174$$
$$752-y-y=174$$
$$752-2y=174$$
$$-2y=-578$$
$$y=289$$

Substitute $y=289$ into ①.

$$x+y=752$$
$$x+289=752$$
$$x=463$$

Section 1: Activity 3 (continued)

Check

LS	RS
$x+y$	752
$=463+289$	
$=752$	

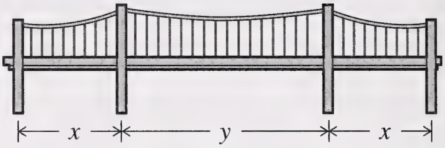
LS = RS

LS	RS
$x-y$	174
$=463-289$	
$=174$	

LS = RS

The two numbers are 463 and 289.

32. Let x be the length of the shorter span, and let y be the length of the longer span.



$$\therefore \quad 2x+y=3560 \qquad ①$$

$$x=\frac{y}{2} \qquad ②$$

Substitute $\frac{y}{2}$ for x into ①.

$$\begin{aligned} 2x+y&=3560 \\ \cancel{2}\left(\frac{y}{\cancel{2}}\right)+y&=3560 \\ y+y&=3560 \\ 2y&=3560 \\ y&=1780 \end{aligned}$$

Substitute $y=1780$ into ②.

$$\begin{aligned} x&=\frac{y}{2} \\ &=\frac{1780}{2} \\ &=890 \end{aligned}$$

Check

LS	RS
$2x+y$	3560
$=2(890)+1780$	
$=1780+1780$	
$=3560$	

LS = RS

LS	RS
x	$\frac{y}{2}$
$=890$	$=\frac{1780}{2}$
	$=890$

LS = RS

The length of each shorter span is 890 m, and the length of the longer span is 1780 m.

33. Let x be the number of hours worked, and let y be the total charged.

$\therefore$ ABC Plumbing: $y=50x+70$ ①

Quality Plumbers: $y=54x+52$ ②

Find the number of hours worked that results in both companies charging the same amount.

Substitute $50x+70$ for y into ②.

$$\begin{aligned} y &= 54x+52 \\ 50x+70 &= 54x+52 \\ -4x &= -18 \\ x &= 4.5 \end{aligned}$$

Substitute $x=4.5$ into ①.

$$\begin{aligned} y &= 50x+70 \\ &= 50(4.5)+70 \\ &= 225+70 \\ &= 295 \end{aligned}$$

Therefore, both ABC Plumbing and Quality Plumbers would charge $295 for 4.5 h of work.

Because Quality Plumbers charge less for a service call, they would be cheaper if the number of hours worked is less than 4.5 h.

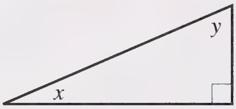
Because ABC Plumbing has a cheaper hourly rate, they would be cheaper if the number of hours worked is greater than 4.5 h.

Section 1: Activity 3 (continued)

35. Let x be the smaller angle, and let y be the other angle.

$\therefore\ y = 3x + 6$ ①

$x + y = 90$ ②



Substitute $3x + 6$ for y into ②.

$$\begin{aligned} x + y &= 90 \\ x + 3x + 6 &= 90 \\ 4x + 6 &= 90 \\ 4x &= 84 \\ x &= 21 \end{aligned}$$

Substitute $x = 21$ into ②.

$$\begin{aligned} x + y &= 90 \\ 21 + y &= 90 \\ y &= 69 \end{aligned}$$

Check

LS	RS
y	$3x + 6$
$= 69$	$= 3(21) + 6$
	$= 63 + 6$
	$= 69$

LS = RS

LS	RS
$x + y$	90
$= 21 + 69$	
$= 90$	

LS = RS

The measure of the smaller angle is 21°, and the measure of the larger angle is 69°.

36. Let x be the number of adult tickets sold, and let y be the number of student tickets sold.

$\therefore\ x + y = 550$ ①

$20x + 12y = 9184$ ②

Solve for x in ①.

$$\begin{aligned} x + y &= 550 \\ x &= 550 - y \end{aligned}$$

Substitute $550 - y$ for x into ②.

$$\begin{aligned} 20x + 12y &= 9184 \\ 20(550 - y) + 12y &= 9184 \\ 11\,000 - 20y + 12y &= 9184 \\ 11\,000 - 8y &= 9184 \\ -8y &= -1816 \\ y &= 227 \end{aligned}$$

Substitute $y = 227$ into ①.

$$\begin{aligned} x + y &= 550 \\ x + 227 &= 550 \\ x &= 323 \end{aligned}$$

Check

LS	RS
$x + y$	550
$= 323 + 227$	
$= 550$	

LS = RS

LS	RS
$20x + 12y$	9184
$= 20(323) + 12(227)$	
$= 6460 + 2724$	
$= 9184$	

LS = RS

There were 323 adult tickets and 227 student tickets sold.

42. Let x be the amount of 60% acid solution, and let $x + 100$ be the amount of 36% acid solution.

Organize the information in a table.

	Acid Solution		
	60% Acid	**30% Acid**	**36% Acid**
Volume of Solution (mL)	x	100	$x + 100$
Volume of Acid (mL)	$0.60x$	$0.30(100)$	$0.36(x + 100)$

Section 1: Activity 3 (continued)

$$\begin{aligned} \therefore\ 0.60x+0.30(100)&=0.36(x+100)\\ 0.60x+30&=0.36x+36\\ 0.24x&=6\\ x&=25 \end{aligned}$$

To make a 36% hydrochloric acid solution, 25 mL of the 60% hydrochloric acid solution must be added.

46. **a.** $x+y+z=3$ ①

$y=4x$ ②

$z=-2x$ ③

Substitute $4x$ for y and $-2x$ for z into ①.

$$\begin{aligned} x+y+z&=3\\ x+4x-2x&=3\\ 3x&=3\\ x&=1 \end{aligned}$$

Substitute $x=1$ into ② and ③.

$$\begin{aligned} y&=4x\\ &=4(1)\\ &=4 \end{aligned} \qquad \begin{aligned} z&=-2x\\ z&=-2(1)\\ z&=-2 \end{aligned}$$

Check

LS	RS
$x+y+z$	3
$=1+4+(-2)$	
$=3$	

LS = RS

LS	RS
y	$4x$
$=4$	$=4(1)$
	$=4$

LS = RS

LS	RS
z	$-2x$
$=-2$	$=-2(1)$
	$=-2$

LS = RS

The solution is $(1,4,-2)$.

b. $2x-3y+z=10$ ①

$x+2z=8$ ②

$y+4z=11$ ③

Solve for x in ② and for y in ③.

$$\begin{aligned} x+2z&=8 \\ x&=8-2z \end{aligned} \qquad \begin{aligned} y+4z&=11 \\ y&=11-4z \end{aligned}$$

Substitute $8-2z$ for x and $11-4z$ for y into ①.

$$\begin{aligned} 2x-3y+z&=10 \\ 2(8-2z)-3(11-4z)+z&=10 \\ 16-4z-33+12z+z&=10 \\ 9z-17&=10 \\ 9z&=27 \\ z&=3 \end{aligned}$$

Substitute $z=3$ into ② and ③.

$$\begin{aligned} x+2z&=8 \\ x+2(3)&=8 \\ x+6&=8 \\ x&=2 \end{aligned} \qquad \begin{aligned} y+4z&=11 \\ y+4(3)&=11 \\ y+12&=11 \\ y&=-1 \end{aligned}$$

Check

LS	RS
$2x-3y+z$	10
$=2(2)-3(-1)+3$	
$=4+3+3$	
$=10$	

LS = RS

LS	RS
$x+2z$	8
$=2+2(3)$	
$=2+6$	
$=8$	

LS = RS

LS	RS
$y+4z$	11
$=-1+4(3)$	
$=-1+12$	
$=11$	

LS = RS

The solution is $(2,-1,3)$.

Section 1: Activity 3 (continued)

9. Textbook questions 1 and 4 of Investigation 1, "Comparing Solubilities," p. 30

1. a. $s = 26 + 0.02T$ ①

$s = 23 + 0.21T$ ②

Substitute $26 + 0.02T$ for s into ②.

$$\begin{aligned} s &= 23 + 0.21T \\ 26 + 0.02T &= 23 + 0.21T \\ -0.19T &= -3 \\ T &\doteq 16 \end{aligned}$$

Their solubilities are equal at approximately 16°C.

b. sodium chloride: $s = 26 + 0.02T$

$\doteq 26 + 0.02(16)$

$\doteq 26 \text{ g}/100 \text{ g}$

ammonium chloride: $s = 23 + 0.21T$

$\doteq 23 + 0.21(16)$

$\doteq 26 \text{ g}/100 \text{ g}$

4. The solubility of sodium chloride, ammonium chloride, and potassium iodide increases as the temperature increases. This is indicated by the positive coefficient of *T*, which corresponds to the slope of the graph.

The solubility of lithium sulfate decreases as the temperature increases. This is indicated by the negative slope in the equation.

Section 1: Activity 4

1. a. Textbook questions a. to d. of "Explore: Use the Equations," p. 34

a. Equation 1 means that the number of tracks on disc 1 and disc 2 totals 60. Equation 2 means that the difference between the number of tracks on disc 1 and disc 2 is 8, with disc 1 having the greater number.

b. $$\begin{array}{r} x + y \\ \underline{x - y} \\ 2x \end{array}$$

c. $$\begin{array}{r} 60 \\ \underline{8} \\ 68 \end{array}$$

d. $2x = 68$

b. Textbook questions 1 to 6 of "Inquire," p. 34

1. $2x = 68$

$x = 34$

2. a. Substitute $x = 34$ into equation 1 or equation 2.

b. $x + y = 60$ or $x - y = 8$

$$\begin{aligned} x + y &= 60 \\ 34 + y &= 60 \\ y &= 26 \end{aligned} \qquad \text{or} \qquad \begin{aligned} x - y &= 8 \\ 34 - y &= 8 \\ -y &= -26 \\ y &= 26 \end{aligned}$$

3. The ordered pair $(34, 26)$ satisfies the system of equations.

4.

LS	RS
$x + y$	60
$= 34 + 26$	
$= 60$	

LS = RS

LS	RS
$x - y$	8
$= 34 - 26$	
$= 8$	

LS = RS

5. There are 34 tracks on disc 1 and 26 tracks on disc 2.

6. a.

$$\begin{aligned} x + y &= 17 \quad ① \\ x - y &= 11 \quad ② \\ \text{Add:} \quad 2x &= 28 \quad ① + ② \\ x &= 14 \end{aligned}$$

Substitute $x = 14$ into ①.

$$\begin{aligned} x + y &= 17 \\ 14 + y &= 17 \\ y &= 3 \end{aligned}$$

Check

LS	RS
$x + y$	17
$= 14 + 3$	
$= 17$	

LS = RS

LS	RS
$x - y$	11
$= 14 - 3$	
$= 11$	

LS = RS

The solution is $(14, 3)$.

Section 1: Activity 4 (continued)

b.

$$\begin{array}{lrl} & x - y = 7 & \textcircled{1} \\ & 2x + y = 20 & \textcircled{2} \\ \hline \text{Add:} & 3x \quad = 27 & \textcircled{1} + \textcircled{2} \\ & x = 9 & \end{array}$$

Substitute $x = 9$ into ①.

$$\begin{aligned} x - y &= 7 \\ 9 - y &= 7 \\ -y &= -2 \\ y &= 2 \end{aligned}$$

Check

LS	RS
$x - y$	7
$= 9 - 2$	
$= 7$	

LS = RS

LS	RS
$2x + y$	20
$= 2(9) + 2$	
$= 18 + 2$	
$= 20$	

LS = RS

The solution is $(9, 2)$.

c.

$$\begin{array}{lrl} & 4x - 3y = 19 & \textcircled{1} \\ & x + 3y = 1 & \textcircled{2} \\ \hline \text{Add:} & 5x \quad = 20 & \textcircled{1} + \textcircled{2} \\ & x = 4 & \end{array}$$

Substitute $x = 4$ into ②.

$$\begin{aligned} x + 3y &= 1 \\ 4 + 3y &= 1 \\ 3y &= -3 \\ y &= -1 \end{aligned}$$

Check

LS	RS
$4x-3y$	19
$=4(4)-3(-1)$	
$=16+3$	
$=19$	

LS = RS

LS	RS
$x+3y$	1
$=4+3(-1)$	
$=4-3$	
$=1$	

LS = RS

The solution is $(4,-1)$.

2. Textbook questions 1, 2, 4, and 6 of "Practice," p. 38

1.

$$\begin{array}{lrl} & 4a-3b=-10 & ① \\ & \underline{2a+3b=\ \ 22} & ② \\ \text{Add:} & 6a \quad = \ \ 12 & ①+② \\ & a=2 & \end{array}$$

Substitute $a=2$ into ②.

$$\begin{aligned} 2a+3b&=22 \\ 2(2)+3b&=22 \\ 4+3b&=22 \\ 3b&=18 \\ b&=6 \end{aligned}$$

Check

LS	RS
$4a-3b$	-10
$=4(2)-3(6)$	
$=8-18$	
$=-10$	

LS = RS

LS	RS
$2a+3b$	22
$=2(2)+3(6)$	
$=4+18$	
$=22$	

LS = RS

The solution is $(2,6)$.

Section 1: Activity 4 (continued)

2.

$$\begin{array}{rrcll} & 5x+2y & = & -11 & ① \\ & 3x+2y & = & -9 & ② \\ \hline \text{Subtract:} & 2x & = & -2 & ① - ② \\ & x & = & -1 & \end{array}$$

Substitute $x=-1$ into ①.

$$\begin{aligned} 5x+2y&=-11 \\ 5(-1)+2y&=-11 \\ -5+2y&=-11 \\ 2y&=-6 \\ y&=-3 \end{aligned}$$

Check

LS	RS
$5x+2y$	-11
$=5(-1)+2(-3)$	
$=-5-6$	
$=-11$	

LS = RS

LS	RS
$3x+2y$	-9
$=3(-1)+2(-3)$	
$=-3-6$	
$=-9$	

LS = RS

The solution is $(-1,-3)$.

4.

$$\begin{array}{rcll} 2m-3n & = & 12 & ① \\ 5m-3n & = & 21 & ② \\ \hline -3m & = & -9 & ① - ② \\ m & = & 3 & \end{array}$$

Substitute $m=3$ into ①.

$$\begin{aligned} 2m-3n&=12 \\ 2(3)-3n&=12 \\ 6-3n&=12 \\ -3n&=6 \\ n&=-2 \end{aligned}$$

Check

LS	RS
$2m-3n$	12
$=2(3)-3(-2)$	
$=6+6$	
$=12$	

LS = RS

LS	RS
$5m-3n$	21
$=5(3)-3(-2)$	
$=15+6$	
$=21$	

LS = RS

The solution is $(3,-2)$.

6. $6y-5x=-7$ ①
$2y-5x=-19$ ②
$4y=12$ ① − ②
$y=3$

Substitute $y=3$ into ②.

$$\begin{aligned} 2y-5x&=-19\\ 2(3)-5x&=-19\\ 6-5x&=-19\\ -5x&=-25\\ x&=5 \end{aligned}$$

Check

LS	RS
$6y-5x$	-7
$=6(3)-5(5)$	
$=18-25$	
$=-7$	

LS = RS

LS	RS
$2y-5x$	-19
$=2(3)-5(5)$	
$=6-25$	
$=-19$	

LS = RS

The solution is $(5,3)$.

Section 1: Activity 4 (continued)

3. a. Textbook questions 9 to 13, 18, 22, 24, 26, and 29 of "Practice," pp. 38 and 39

9. $4x+3y=15$ ①

$8x-9y=15$ ②

Multiply ① by 2: $8x+6y=30$ ③ : 2×①

$8x-9y=15$ ②

Subtract: $15y=15$ ③ − ②

$y=1$

Substitute $y=1$ into ①.

$$\begin{aligned}4x+3y&=15\\4x+3(1)&=15\\4x+3&=15\\4x&=12\\x&=3\end{aligned}$$

Check

LS	RS
$4x+3y$	15
$=4(3)+3(1)$	
$=12+3$	
$=15$	

LS = RS

LS	RS
$8x-9y$	15
$=8(3)-9(1)$	
$=24-9$	
$=15$	

LS = RS

The solution is $(3,1)$.

10. $3r+2s=5$ ①

$9r+6s=7$ ②

Multiply ① by 3: $9r+6s=15$ ③ : 3×①

$9r+6s=\ 7$ ②

Subtract: $0=\ 8$ ③ − ②

The system has no solution. The system consists of two parallel lines with different *y*-intercepts.

11. $2x-3y=2$ ①
$5x+6y=5$ ②

Multiply ① by 2: $4x-6y=4$ ③ : 2× ①
$5x+6y=5$ ②

Add: $9x\quad=9$ ③ + ②
$x=1$

Substitute $x=1$ into ①.

$$\begin{aligned}2x-3y&=2\\2(1)-3y&=2\\2-3y&=2\\-3y&=0\\y&=0\end{aligned}$$

Check

LS	RS
$2x-3y$	2
$=2(1)-3(0)$	
$=2$	

LS = RS

LS	RS
$5x+6y$	5
$=5(1)+6(0)$	
$=5$	

LS = RS

The solution is $(1,0)$.

12. $4x-3y=5$ ①
$8x-6y=10$ ②

Multiply ① by 2: $8x-6y=10$ ③ : 2× ①
$8x-6y=10$ ②

Subtract: $0=0$ ③ − ②

This system has an infinite number of solutions. The system consists of two equations that represent the same line.

Section 1: Activity 4 (continued)

13. $3a+2b=16$ ①

$2a+3b=14$ ②

Multiply ① by 2: $6a+4b=32$ ③ : 2×①

Multiply ② by 3: $\underline{6a+9b=42}$ ④ : 3×②

Subtract: $-5b=-10$ ③ – ④

$b=2$

Substitute $b=2$ into ①.

$$\begin{aligned}3a+2b&=16\\3a+2(2)&=16\\3a+4&=16\\3a&=12\\a&=4\end{aligned}$$

Check

LS	RS
$3a+2b$	16
$=3(4)+2(2)$	
$=12+4$	
$=16$	

LS = RS

LS	RS
$2a+3b$	14
$=2(4)+3(2)$	
$=8+6$	
$=14$	

LS = RS

The solution is $(4,2)$.

18. $6x+5y=22$

$3y=4x+36$ $\xrightarrow{\text{Rearrange}}$ $6x+5y=22$ ①

$-4x+3y=36$ ②

Rearrange equations so similar terms always line up.

$12x+10y=44$ ③ : 2×①

$\underline{-12x+9y=108}$ ④ : 3×②

$19y=152$ ③ + ④

$y=8$

Substitute $y=8$ into ①.

$$\begin{aligned} 6x+5y&=22 \\ 6x+5(8)&=22 \\ 6x+40&=22 \\ 6x&=-18 \\ x&=-3 \end{aligned}$$

Check

LS	RS
$6x+5y$	22
$=6(-3)+5(8)$	
$=-18+40$	
$=22$	

LS $=$ RS

LS	RS
$3y$	$4x+36$
$=3(8)$	$=4(-3)+36$
$=24$	$=-12+36$
	$=24$

LS $=$ RS

The solution is $(-3,8)$.

22.

$$\begin{aligned} 3s+4&=-4t \\ 7s+6t+11&=0 \end{aligned} \xrightarrow{\text{Rearrange}} \begin{aligned} 3s+4t&=-4 \quad \text{①} \\ 7s+6t&=-11 \quad \text{②} \end{aligned}$$

$$\begin{array}{rcll} 9s+12t &=& -12 & \text{③} : 3\times\text{①} \\ 14s+12t &=& -22 & \text{④} : 2\times\text{②} \\ \hline -5s &=& 10 & \text{③}-\text{④} \\ s &=& -2 & \end{array}$$

Substitute $s=-2$ into ①.

$$\begin{aligned} 3s+4t&=-4 \\ 3(-2)+4t&=-4 \\ -6+4t&=-4 \\ 4t&=2 \\ t&=\frac{2}{4} \\ &=\frac{1}{2} \end{aligned}$$

Section 1: Activity 4 (continued)

Check

LS	RS
$3s+4$	$-4t$
$=3(-2)+4$	$=\overset{-2}{\cancel{-4}}\left(\dfrac{1}{\underset{1}{\cancel{2}}}\right)$
$=-6+4$	
$=-2$	$=-2$

LS $=$ RS

LS	RS
$7s+6t+11$	0
$=7(-2)+\overset{3}{\cancel{6}}\left(\dfrac{1}{\underset{1}{\cancel{2}}}\right)+11$	
$=-14+3+11$	
$=0$	

LS $=$ RS

The solution is $\left(-2, \frac{1}{2}\right)$.

24.

$$\begin{array}{rl} 2d=10+4e & \text{①} \\ 3d=15+6e & \text{②} \end{array}$$

$$\begin{array}{rl} 6d=30+12e & \text{③}: 3\times\text{①} \\ 6d=30+12e & \text{④}: 2\times\text{②} \\ \hline 0=0 & \text{③}-\text{④} \end{array}$$

Therefore, there is an infinite number of solutions because both equations represent the same line.

26.

$$\begin{array}{l} 4x-5=2y \\ 1=5y-10x \end{array} \xrightarrow{\text{Rearrange}} \begin{array}{rl} 4x-5=2y & \text{①} \\ 10x+1=5y & \text{②} \end{array}$$

$$\begin{array}{rl} 20x-25=10y & \text{③}: 5\times\text{①} \\ 20x+\ \ 2=10y & \text{④}: 2\times\text{②} \\ \hline -27=0 & \text{③}-\text{④} \end{array}$$

No ordered pair satisfies this system of equations; thus, there is no solution.

29. $2(a-4)+5(b+1)=8$ ①
$3(a-1)-2(b-2)=-11$ ②

Simplify ① and ②.

$$\begin{aligned}2(a-4)+5(b+1)&=8\\2a-8+5b+5&=8\\2a+5b&=11\end{aligned}\qquad\qquad\begin{aligned}3(a-1)-2(b-2)&=-11\\3a-3-2b+4&=-11\\3a-2b&=-12\end{aligned}$$

Rewrite the system of equations, and solve.

$2a+5b=11$ ①
$3a-2b=-12$ ②

$6a+15b=\ 33$ ③ : 3×①
$6a-\ 4b=-24$ ④ : 2×②
$19b=\ 57$ ③ − ④
$b=3$

Substitute $b=3$ into ①.

$$\begin{aligned}2a+5b&=11\\2a+5(3)&=11\\2a+15&=11\\2a&=-4\\a&=-2\end{aligned}$$

Check

LS	RS
$2(a-4)+5(b+1)$	8
$=2(-2-4)+5(3+1)$	
$=2(-6)+5(4)$	
$=-12+20$	
$=8$	

LS = RS

LS	RS
$3(a-1)-2(b-2)$	-11
$=3(-2-1)-2(3-2)$	
$=3(-3)-2(1)$	
$=-9-2$	
$=-11$	

LS = RS

The solution is $(-2,3)$.

Section 1: Activity 4 (continued)

b. Textbook questions 43, 45, and 50 of "Applications and Problem Solving," pp. 39 and 40

43. a. For $a+b=10$, the number of provinces that have names with First Nations origins plus the number of provinces with other names is 10. The total number of provinces is 10.

For $3a-2b=0$, three times the number of provinces that have names with First Nations origins equals two times the number of provinces with other names.

b.

$a+b=10$ ①

$3a-2b=0$ ②

$3a+3b=30$ ③ : 3 × ①

$3a-2b=\ 0$ ②

$5b=30$ ③ − ②

$b=6$

Substitute $b=6$ into ①.

$$a+b=10$$
$$a+6=10$$
$$a=4$$

Check

LS	RS
$a+b$	10
$=4+6$	
$=10$	

LS = RS

LS	RS
$3a-2b$	0
$=3(4)-2(6)$	
$=12-12$	
$=0$	

LS = RS

The number of provinces that have names with First Nations origins is 4.

45. $2x+3y=124$ ① ← Opposite angles are equal.

$3x-2y+124=180$ ② ← The adjacent angles are supplementary.

$2x+3y=124$ ①

$3x-2y=56$ ② ← Simplify.

$4x+6y=248$ ③ : 2×①

$9x-6y=168$ ④ : 3×②

$13x=416$ ③ + ④

$x=32$

Substitute $x=32$ into ①.

$$\begin{aligned} 2x+3y&=124 \\ 2(32)+3y&=124 \\ 64+3y&=124 \\ 3y&=60 \\ y&=20 \end{aligned}$$

Check

LS	RS
$2x+3y$	124
$=2(32)+3(20)$	
$=64+60$	
$=124$	

LS = RS

LS	RS
$3x-2y+124$	180
$=3(32)-2(20)+124$	
$=96-40+124$	
$=180$	

LS = RS

The solution is $(32, 20)$.

Section 1: Activity 4 (continued)

50. Let x be the price of each chicken sandwich, and let y be the price of each cheese sandwich.

$$\therefore\ 2x+4y=18 \quad ①$$
$$5x+6y=34 \quad ②$$

$$\begin{array}{rcll} 6x+12y & = & 54 & ③ : 3\times ① \\ 10x+12y & = & 68 & ④ : 2\times ② \\ \hline -4x & = & -14 & ③ - ④ \\ x & = & 3.50 & \end{array}$$

Substitute $x=3.50$ into ①.

$$\begin{aligned} 2(3.50)+4y &= 18 \\ 7+4y &= 18 \\ 4y &= 11 \\ y &= 2.75 \end{aligned}$$

Check

LS	RS
$2x+4y$	18
$=2(3.50)+4(2.75)$	
$=7+11$	
$=18$	

LS = RS

LS	RS
$5x+6y$	34
$=5(3.50)+6(2.75)$	
$=17.50+16.50$	
$=34$	

LS = RS

The cost of a chicken sandwich is \$3.50, and the cost of the cheese sandwich is \$2.75; therefore, the chicken is \$0.75 or 75¢ more expensive than the cheese sandwich.

4. a. Textbook questions 31 and 35 of "Practice," p. 39

31. $\frac{x}{3}+\frac{y}{4}=2$ ①

$\frac{2x}{3}-\frac{y}{2}=0$ ②

$$\begin{array}{rl}
4x+3y=24 & ③ : 12\times ① \\
4x-3y=\ \ 0 & ④ : 6\times ② \\
\hline
8x \quad\ \ =24 & ③ + ④ \\
x=3 &
\end{array}$$

Substitute $x=3$ into ③.

$$\begin{aligned}
4x+3y&=24\\
4(3)+3y&=24\\
12+3y&=24\\
3y&=12\\
y&=4
\end{aligned}$$

Check

LS	RS
$\frac{x}{3}+\frac{y}{4}$	2
$=\frac{3}{3}+\frac{4}{4}$	
$=1+1$	
$=2$	

LS = RS

LS	RS
$\frac{2x}{3}-\frac{y}{2}$	0
$=\frac{2(\cancel{3})}{\cancel{3}}-\frac{4}{2}$	
$=2-2$	
$=0$	

LS = RS

The solution is $(3,4)$.

Section 1: Activity 4 (continued)

35. $0.3x-0.5y=1.2$ ①
$0.7x-0.2y=-0.1$ ②

$3x-5y=12$ ③ : 10×①
$7x-2y=-1$ ④ : 10×②

$$\begin{array}{rcl} 6x-10y &=& 24 \\ 35x-10y &=& -5 \\ \hline -29x \qquad &=& 29 \\ x &=& -1 \end{array}$$

⑤ : 2×③
⑥ : 5×④
⑤ – ⑥

Substitute $x=-1$ into ③.

$$\begin{aligned} 3x-5y&=12 \\ 3(-1)-5y&=12 \\ -3-5y&=12 \\ -5y&=15 \\ y&=-3 \end{aligned}$$

Check

LS	RS
$0.3x-0.5y$	1.2
$=0.3(-1)-0.5(-3)$	
$=-0.3+1.5$	
$=1.2$	

LS = RS

LS	RS
$0.7x-0.2y$	-0.1
$=0.7(-1)-0.2(-3)$	
$=-0.7+0.6$	
$=-0.1$	

LS = RS

The solution is $(-1,-3)$.

b. Textbook questions 44 and 46 of "Applications and Problem Solving," p. 39

44. $\frac{a}{2}+\frac{b}{5}=173$ ①

$\frac{a}{3}+\frac{b}{6}=127$ ②

$5a+2b=1730$ ③ : $10\times$①

$2a+b=762$ ④ : $6\times$②

$5a+2b=1730$ ③

$\underline{4a+2b=1524}$ ⑤ : $2\times$④

$a \quad = 206$ ③ − ⑤

Substitute $a=206$ into ④.

$$\begin{aligned} 2a+b&=762\\ 2(206)+b&=762\\ 412+b&=762\\ b&=350 \end{aligned}$$

Check

LS	RS
$\frac{a}{2}+\frac{b}{5}$	173
$=\frac{206}{2}+\frac{350}{5}$	
$=103+70$	
$=173$	

LS = RS

LS	RS
$\frac{a}{3}+\frac{b}{6}$	127
$=\frac{206}{3}+\frac{350}{6}$	
$=\frac{206}{3}+\frac{175}{3}$	
$=\frac{381}{3}$	
$=127$	

LS = RS

The average number of bones for an adult is 206, and the average number of bones for a baby is 350.

Section 1: Activity 4 (continued)

46. Let x be one number, and let y be the other number.

$$\therefore \quad \frac{x+y}{2}=5 \qquad ①$$

$$4x+3y=2 \qquad ②$$

$$x+y=10 \qquad ③: 2\times①$$

$$4x+3y=2 \qquad ②$$

$$\begin{array}{rl} 3x+3y=30 & ④: 3\times② \\ 4x+3y=\ \ 2 & ② \\ \hline -x \qquad =28 & ④-② \\ x=-28 & \end{array}$$

Substitute $x=-28$ into ③.

$$\begin{aligned} x+y&=10 \\ -28+y&=10 \\ y&=38 \end{aligned}$$

Check

LS	RS
$\frac{x+y}{2}$	5
$=\frac{-28+38}{2}$	
$=\frac{10}{2}$	
$=5$	

LS = RS

LS	RS
$4x+3y$	2
$=4(-28)+3(38)$	
$=-112+114$	
$=2$	

LS = RS

The two numbers are -28 and 38.

5. Textbook questions 47 and 48 of "Applications and Problem Solving," p. 39

47. Let b represent the speed of the boat in still water, and let c represent the speed of the current.

Therefore, $b-c$ is the speed of the boat travelling upstream and $b+c$ is the speed of the boat travelling downstream.

Use the formula distance = speed × time, and use a table to organize the information.

Direction	Distance (km)	Speed (km/h)	Time (h)	Equation
Upstream	60	$b-c$	5	$5(b-c)=60$
Downstream	60	$b+c$	3	$3(b+c)=60$

$$\therefore\ 5b-5c=60 \quad ①$$
$$3b+3c=60 \quad ②$$

$$\begin{aligned}15b-15c&=180 && ③ : 3\times ①\\ 15b+15c&=300 && ④ : 5\times ②\\ \hline 30b\qquad&=480 && ③+④\\ b&=16\end{aligned}$$

Substitute $b=16$ into ②.

$$\begin{aligned}3b+3c&=60\\ 3(16)+3c&=60\\ 48+3c&=60\\ 3c&=12\\ c&=4\end{aligned}$$

Check

LS	RS
$5(b-c)$ $=5(16-4)$ $=5(12)$ $=60$	60

LS = RS

LS	RS
$3(b+c)$ $=3(16+4)$ $=3(20)$ $=60$	60

LS = RS

The speed of the boat in still water is 16 km/h, and the speed of the current is 4 km/h.

Section 1: Activity 4 (continued)

48. Let x be the speed of the plane in still air, and let y be the speed of the wind.

Therefore, $x+y$ is the speed of the plane flying with a tail wind and $x-y$ is the speed of the plane flying with a head wind.

Use the formula distance = speed × time, and use a table to organize the information.

Direction	Distance (km)	Speed (km/h)	Time (h)	Equation
With Tail Wind	2200	$x+y$	4	$4(x+y)=2200$
With Head Wind	2200	$x-y$	5	$5(x-y)=2200$

$\therefore\ 4x+4y=2200$ ①

$5x-5y=2200$ ②

$20x+20y=11\,000$ ③ : 5 × ①

$20x-20y=\ 8\,800$ ④ : 4 × ②

$40x\ =19\,800$ ③ + ④

$x=495$

Substitute $x=495$ into ①.

$$\begin{aligned}4x+4y&=2200\\4(495)+4y&=2200\\1980+4y&=2200\\4y&=220\\y&=55\end{aligned}$$

Check

LS	RS
$4(x+y)$	2200
$=4(495+55)$	
$=4(550)$	
$=2200$	

LS = RS

LS	RS
$5(x-y)$	2200
$=5(495-55)$	
$=5(440)$	
$=2200$	

LS = RS

The speed of the plane in still air is 495 km/h, and the speed of the wind is 55 km/h.

6. Textbook questions 49, 51, 52, 56, 57, and 58.a. of “Applications and Problem Solving,” pp. 39 and 40

49. Let x be the distance travelled at 90 km/h, and let y be the distance travelled at 100 km/h.

Therefore, the time while travelling 90 km/h is $\frac{x}{90}$, and the time while travelling 100 km/h is $\frac{y}{100}$.

Remember:

distance = speed × time

$time = \frac{distance}{speed}$

$\therefore\ x + y = 470$ ①

$\frac{x}{90} + \frac{y}{100} = 5$ ②

$10x + 10y = 4700$ ③ : 10 × ①

$10x + 9y = 4500$ ④ : 900 × ②

$y = 200$ ③ − ④

Substitute $y = 200$ into ①.

$$x + y = 470$$
$$x + 200 = 470$$
$$x = 270$$

Check

LS	RS
$x + y$	470
$= 200 + 270$	
$= 470$	

LS = RS

LS	RS
$\frac{x}{90} + \frac{y}{100}$	5
$= \frac{270}{90} + \frac{200}{100}$	
$= 3 + 2$	
$= 5$	

LS = RS

Therefore, Kareem drove 270 km at 90 km/h and 200 km at 100 km/h.

Section 1: Activity 4 (continued)

51. Let ℓ be the length, and let w be the width.

$\therefore \quad 2\ell + 2w = 7.8$ ①

$4\ell = 9w$ ②

$4\ell + 4w = 15.6$ ③: 2 × ①

$4\ell - 9w = 0.0$ ② ← Rearrange.

$13w = 15.6$ ③ – ②

$w = 1.2$

Substitute $w = 1.2$ into ②.

$$\begin{aligned} 4\ell &= 9w \\ 4\ell &= 9(1.2) \\ 4\ell &= 10.8 \\ \ell &= 2.7 \end{aligned}$$

Check

LS	RS
$2\ell + 2w$	7.8
$= 2(2.7) + 2(1.2)$	
$= 5.4 + 2.4$	
$= 7.8$	

LS = RS

LS	RS
4ℓ	$9w$
$= 4(2.7)$	$= 9(1.2)$
$= 10.8$	$= 10.8$

LS = RS

The dimensions of the pool table are $2.7 \text{ m} \times 1.2 \text{ m}$.

52. Let x be the amount of time playing tennis, and let y be the amount of time cycling.

$\therefore\ x+y=50$ ①

$25x+35y=1450$ ②

$25x+25y=1250$ ③: 25×①

$25x+35y=1450$ ②

$-10y=-200$ ③ − ②

$y=20$

Substitute $y=20$ into ①.

$$\begin{aligned} x+y&=50 \\ x+20&=50 \\ x&=30 \end{aligned}$$

Check

LS	RS
$x+y$	50
$=30+20$	
$=50$	

LS = RS

LS	RS
$25x+35y$	1450
$=25(30)+35(20)$	
$=750+700$	
$=1450$	

LS = RS

Hans spent 30 min playing tennis and 20 min cycling.

56. Substitute $(5,-3)$ and $(-3,9)$ into the equation.

$$\begin{aligned} Ax+By-9&=0 \\ A(5)+B(-3)&=9 \\ 5A-3B&=9 \end{aligned}$$ ①

$$\begin{aligned} Ax+By-9&=0 \\ A(-3)+B(9)&=9 \\ -3A+9B&=9 \end{aligned}$$ ②

$15A-9B=27$ ③: 3×①

$-3A+9B=9$ ②

$12A=36$ ③ + ②

$A=3$

Section 1: Activity 4 (continued)

Substitute $A = 3$ into ①.

$$\begin{aligned} 5A - 3B &= 9 \\ 5(3) - 3B &= 9 \\ 15 - 3B &= 9 \\ -3B &= -6 \\ B &= 2 \end{aligned}$$

Check

For $(5, -3)$,

LS	RS
$Ax + By - 9$	0
$= 5(3) - 3(2) - 9$	
$= 15 - 6 - 9$	
$= 0$	

LS = RS

For $(-3, 9)$,

LS	RS
$Ax + By - 9$	0
$= -3(3) + 9(2) - 9$	
$= -9 + 18 - 9$	
$= 0$	

LS = RS

Therefore $A = 3$ and $B = 2$.

57. Substitute $(2, -1)$ into both equations.

$$\begin{aligned} ax + by &= -7 \\ a(2) + b(-1) &= -7 \\ 2a - b &= -7 \quad ① \end{aligned}$$

$$\begin{aligned} 2ax - 3by &= 1 \\ 2a(2) - 3b(-1) &= 1 \\ 4a + 3b &= 1 \quad ② \end{aligned}$$

$$\begin{aligned} 4a - 2b &= -14 && ③ : 2 \times ① \\ \underline{4a + 3b} &\underline{{}= \quad 1} && ② \\ -5b &= -15 && ③ - ② \\ b &= 3 \end{aligned}$$

Substitute $b = 3$ into ①.

$$\begin{aligned} 2a - b &= -7 \\ 2a - 3 &= -7 \\ 2a &= -4 \\ a &= -2 \end{aligned}$$

Check

LS	RS
$ax+by$	-7
$=-2(2)+3(-1)$	
$=-4-3$	
$=-7$	

LS $=$ RS

LS	RS
$2ax-3by$	1
$=2(-2)(2)-3(3)(-1)$	
$=-8+9$	
$=1$	

LS $=$ RS

Therefore, $a=-2$ and $b=3$.

58. **a.** $2x-6y=c$ ①

$6x-18y=30$ ②

$6x-18y=3c$ ③ : $3\times$①

$6x-18y=30$ ②

$0=3c-30$ ③ $-$ ②

For the system to have an infinite number of solutions, the difference of the (equivalent) equations must be of the form $0=0$.

$\therefore\ 0=3c-30$

$-3c=-30$

$c=10$

Check

$2x-6y=10$ ①

$6x-18y=30$ ②

$6x-18y=30$ ③ : $3\times$①

$6x-18y=30$ ②

$0=\ 0$ ③ $-$ ②

Note: Both equations have an identical slope and y-intercept form, $y=mx+b$. This indicates that the graphs of the equations coincide.

Section 1: Activity 4 (continued)

7. **Textbook question 2 of "Applications and Problem Solving," p. 49**

   2. Answers may vary slightly.

   The volume of 1 coin can be determined by using a graduated cylinder to observe the rise in water level when the coin is submerged. For a more accurate result, submerge 10 coins.

   The volume of 10^1 coins is about 8 mL. Therefore, the volume of 10^6 coins is 8×10^5 mL, which is equal to $\frac{8\times10^5\text{ mL}}{10^3\text{ mL/L}} = 8\times10^2\text{ L}$.

   The volume of one million loonies is approximately 800 L.

8. Answers may vary slightly.

The population of the world is approximately 6 billion, or 6×10^9. This requires $6\times10^9\text{ m}^2$.

Alberta's surface area is approximately $6.6\times10^5\text{ km}^2$, which is approximately $6.6\times10^5\text{ km}^2 \times \frac{10^6\text{ m}^2}{1\text{ km}^2} = 6.6\times10^{11}\text{ m}^2$.

$$\begin{aligned}\text{Percentage of Surface Area} &\doteq \frac{6\times10^9\text{ m}^2}{6.6\times10^{11}\text{ m}^2}\times100\% \\ &\doteq 0.9\%\end{aligned}$$

Less than 1% of Alberta's area would be needed to provide standing room for everyone on Earth.

Section 1: Activity 5

1. **Textbook questions 1 to 5, 6.b., and 7.b. of "Inquire," p. 41**

   1. a. When ① and ② are added, w is eliminated.

   b.

$$\begin{array}{rl} c+e+w=28 & ① \\ c+e-w=4 & ② \\ \hline 2c+2e \quad =32 & ①+② \end{array}$$

2. **a.** When ② and ③ are added, w is eliminated.

b.

$$\begin{array}{rl} c+e-w=4 & \text{②} \\ c-2e+w=10 & \text{③} \\ \hline 2c-e\quad=14 & \text{②}+\text{③} \end{array}$$

3.

$$\begin{array}{rl} 2c+2e=32 & \text{④}:\text{①}+\text{②} \\ 2c-e=14 & \text{⑤}:\text{②}+\text{③} \\ \hline 3e=18 & \text{④}-\text{⑤} \\ e=6 & \end{array}$$

Substitute $e=6$ into ⑤.

$$\begin{aligned} 2c-e&=14 \\ 2c-6&=14 \\ 2c&=20 \\ c&=10 \end{aligned}$$

4. **a.** The variable w can be found from any one of the three equations by substituting the values of e and c. The equation can then be solved for w.

b. Substitute $c=10$ and $e=6$ into ①.

$$\begin{aligned} c+e+w&=28 \\ 10+6+w&=28 \\ 16+w&=28 \\ w&=12 \end{aligned}$$

5. There are 10 major public art galleries in Central Canada, 6 in Eastern Canada, and 12 in Western Canada.

6. **b.**

LS	RS
$c+e+w$	28
$=10+6+12$	
$=28$	

LS = RS

LS	RS
$c+e-w$	4
$=10+6-12$	
$=4$	

LS = RS

LS	RS
$c-2e+w$	10
$=10-2(6)+12$	
$=10-12+12$	
$=10$	

LS = RS

Section 1: Activity 5 (continued)

7. **b.** $2a-b+c=1$ ①

$a+b+c=8$ ②

$a-b+c=0$ ③

$a+b+c=8$ ②

$\underline{a-b+c=0}$ ③

$2b\quad=8$ ② − ③

$b=4$

Substitute $b=4$ into ① and ②.

$2a-b+c=1$

$2a-4+c=1$

$2a+c=5$ ④

$a+b+c=8$

$a+4+c=8$

$a+c=4$ ⑤

$2a+c=5$ ④

$\underline{a+c=4}$ ⑤

$a\quad=1$ ④ − ⑤

Substitute $a=1$ into ⑤.

$a+c=4$

$1+c=4$

$c=3$

Check

LS	RS
$2a-b+c$	1
$=2(1)-4+3$	
$=1$	

LS = RS

LS	RS
$a+b+c$	8
$=1+4+3$	
$=8$	

LS = RS

LS	RS
$a-b+c$	0
$=1-4+3$	
$=0$	

LS = RS

The solution is $a=1$, $b=4$, and $c=3$.

2. Textbook questions 1, 2, 6, 8, and 10 of "Practice," p. 44

1.

LS	RS
$3x+y-z$	7
$=3(2)+2-1$	
$=6+2-1$	
$=7$	

LS $=$ RS

The ordered triple is a solution.

2.

LS	RS
$4x-y-z$	8
$=4(3)-(-1)-(4)$	
$=12+1-4$	
$=9$	

LS $\neq$ RS

The ordered triple is not a solution.

6.

LS	RS
$3a-3b-c$	-6
$=3(-2)-3(1)-(-2)$	
$=-6-3+2$	
$=-7$	

LS $\neq$ RS

The ordered triple is not a solution.

Section 1: Activity 5 (continued)

8.

LS	RS
$4a-3b-2c$	31
$=4(3)-3(-5)-2(-2)$	
$=12+15+4$	
$=31$	

LS = RS

LS	RS
$7a-3b+5c$	26
$=7(3)-3(-5)+5(-2)$	
$=21+15-10$	
$=26$	

LS = RS

LS	RS
$5a-b-4c$	28
$=5(3)-(-5)-4(-2)$	
$=15+5+8$	
$=28$	

LS = RS

Because $(3,-5,-2)$ satisfies every equation of the system, it is a solution of the system.

10.

LS	RS
$3u+v+w$	3
$=3\left(\frac{1}{2}\right)+2+\left(-\frac{1}{2}\right)$	
$=\frac{3}{2}+2-\frac{1}{2}$	
$=3$	

LS = RS

LS	RS
$u-v-w$	-2
$=\frac{1}{2}-2-\left(-\frac{1}{2}\right)$	
$=\frac{1}{2}-2+\frac{1}{2}$	
$=-1$	

LS ≠ RS

LS	RS
$2u+3v-4w$	10
$=\cancel{2}\left(\frac{1}{\cancel{2}}\right)+3(2)-\overset{2}{\cancel{4}}\left(-\frac{1}{\underset{1}{\cancel{2}}}\right)$	
$=1+6+2$	
$=9$	

LS ≠ RS

Because $\left(\frac{1}{2},2,-\frac{1}{2}\right)$ does not satisfy every equation of the system, it is not a solution of the system.

3. a. Textbook questions 12, 14, 16, and 26 of "Practice," p. 44

12. $x+y+z=-1$ ①

$2x+3y-z=5$ ②

$3x-2y-z=0$ ③

$$\begin{array}{rl} x+y+z=-1 & ① \\ 2x+3y-z=5 & ② \\ \hline 3x+4y=4 & ④ : ① + ② \end{array}$$

$$\begin{array}{rl} x+y+z=-1 & ① \\ 3x-2y-z=0 & ③ \\ \hline 4x-y=-1 & ⑤ : ① + ③ \end{array}$$

$$\begin{array}{rl} 3x+4y=4 & ④ \\ 16x-4y=-4 & ⑥ : 4\times ④ \\ \hline 19x=0 & ④ + ⑥ \\ x=0 & \end{array}$$

Substitute $x=0$ into ④.

$$\begin{aligned} 3x+4y&=4 \\ 3(0)+4y&=4 \\ 4y&=4 \\ y&=1 \end{aligned}$$

Substitute $x=0$ and $y=1$ into ①.

$$\begin{aligned} x+y+z&=-1 \\ 0+1+z&=-1 \\ z&=-2 \end{aligned}$$

Check

LS	RS
$x+y+z$	-1
$=0+1+(-2)$	
$=-1$	

LS = RS

LS	RS
$2x+3y-z$	5
$=2(0)+3(1)-(-2)$	
$=3+2$	
$=5$	

LS = RS

Section 1: Activity 5 (continued)

LS	RS
$3x-2y-z$	0
$=3(0)-2(1)-(-2)$	
$=-2+2$	
$=0$	

$$\text{LS} = \text{RS}$$

The solution is $x=0$, $y=1$, and $z=-2$, or $(0,1,-2)$.

14.
$$\begin{aligned} e+f+g&=-6 && ① \\ e-2f-2g&=9 && ② \\ e+3f+4g&=-19 && ③ \end{aligned}$$

$$\begin{aligned} e+\ f+\ g&=\ -6 && ① \\ e-2f-2g&=\ \ \ 9 && ② \\ \hline 3f+3g&=-15 && ①-② \\ f+g&=-5 && ④ \end{aligned}$$

$$\begin{aligned} e+\ f+\ g&=\ -6 && ① \\ e+3f+4g&=-19 && ③ \\ \hline -2f-3g&=\ 13 && ⑤:①-③ \end{aligned}$$

$$\begin{aligned} 2f+2g&=-10 && ⑥: 2\times ④ \\ -2f-3g&=\ 13 && ⑤ \\ \hline -g&=\ \ 3 && ⑥+⑤ \\ g&=-3 \end{aligned}$$

Substitute $g=-3$ into ④.

$$\begin{aligned} f+g&=-5 \\ f+(-3)&=-5 \\ f&=-2 \end{aligned}$$

Substitute $f=-2$ and $y=-3$ into ①.

$$\begin{aligned} e+f+g&=-6 \\ e+(-2)+(-3)&=-6 \\ e-5&=-6 \\ e&=-1 \end{aligned}$$

Check

$$\begin{array}{l|l}
\text{LS} & \text{RS} \\
\hline
e+f+g & -6 \\
=-1+(-2)+(-3) & \\
=-6 & \\
\end{array}$$

$$\text{LS} = \text{RS}$$

$$\begin{array}{l|l}
\text{LS} & \text{RS} \\
\hline
e-2f-2g & 9 \\
=-1-2(-2)-2(-3) & \\
=-1+4+6 & \\
=9 & \\
\end{array}$$

$$\text{LS} = \text{RS}$$

$$\begin{array}{l|l}
\text{LS} & \text{RS} \\
\hline
e+3f+4g & -19 \\
=-1+3(-2)+4(-3) & \\
=-1-6-12 & \\
=-19 & \\
\end{array}$$

$$\text{LS} = \text{RS}$$

The solution is $e=-1$, $f=-2$, and $g=-3$, or $(-1,-2,-3)$.

16. $a+2b+c=0$ ①

$3a-b-2c=11$ ②

$2a+b-c=3$ ③

$$\begin{array}{rl}
a+2b+\ \ c=\ \ 0 & ① \\
6a-2b-4c=22 & ④: 2\times ② \\
\hline
7a \qquad -3c=22 & ⑤: ① + ④
\end{array}
\qquad
\begin{array}{rl}
3a-b-2c=11 & ② \\
2a+b-\ \ c=\ \ 3 & ③ \\
\hline
5a \qquad -3c=14 & ⑥: ② + ③
\end{array}$$

$$\begin{array}{rl}
7a-3c=22 & ⑤ \\
5a-3c=14 & ⑥ \\
\hline
2a \qquad =\ 8 & ⑤ - ⑥ \\
a=4 &
\end{array}$$

Section 1: Activity 5 (continued)

Substitute $a=4$ into ⑤.

$$\begin{aligned} 7a-3c&=22\\ 7(4)-3c&=22\\ 28-3c&=22\\ -3c&=-6\\ c&=2 \end{aligned}$$

Substitute $a=4$ and $c=2$ into ①.

$$\begin{aligned} a+2b+c&=0\\ 4+2b+2&=0\\ 2b&=-6\\ b&=-3 \end{aligned}$$

Check

LS	RS
$a+2b+c$	0
$=4+2(-3)+2$	
$=4-6+2$	
$=0$	

LS = RS

LS	RS
$3a-b-2c$	11
$=3(4)-(-3)-2(2)$	
$=12+3-4$	
$=11$	

LS = RS

LS	RS
$2a+b-c$	3
$=2(4)+(-3)-2$	
$=8-3-2$	
$=3$	

LS = RS

The solution is $a=4$, $b=-3$, and $c=2$, or $(4,-3,2)$.

26. $3x-y+2z=4$ ①

$2x+2y-z=19$ ②

$4x-2y-3z=35$ ③

$$\begin{array}{rcll} 6x-2y+4z &=& 8 & \text{④ : } 2\times\text{①} \\ 2x+2y-z &=& 19 & \text{②} \\ \hline 8x+3z &=& 27 & \text{⑤ : ④ + ②} \end{array}$$

$$\begin{array}{rcll} 2x+2y-z &=& 19 & \text{②} \\ 4x-2y-3z &=& 35 & \text{③} \\ \hline 6x-4z &=& 54 & \text{② + ③} \\ 3x-2z &=& 27 & \text{⑥} \end{array}$$

$$\begin{array}{rcll} 16x+6z &=& 54 & \text{⑦ : } 2\times\text{⑤} \\ 9x-6z &=& 81 & \text{⑧ : } 3\times\text{⑥} \\ \hline 25x &=& 135 & \text{⑦ + ⑧} \end{array}$$

$$\begin{aligned} x &= \frac{135}{25} \\ &= \frac{27}{5} \end{aligned}$$

Substitute $x=\frac{27}{5}$ into ⑤.

$$\begin{aligned} 8x+3z &= 27 \\ 8\left(\frac{27}{5}\right)+3z &= 27 \\ \frac{216}{5}+3z &= \frac{135}{5} \\ 3z &= -\frac{81}{5} \\ z &= -\frac{27}{5} \end{aligned}$$

Substitute $x=\frac{27}{5}$ and $z=-\frac{27}{5}$ into ①.

$$\begin{aligned} 3x-y+2z &= 4 \\ 3\left(\frac{27}{5}\right)-y+2\left(-\frac{27}{5}\right) &= 4 \\ \frac{81}{5}-y-\frac{54}{5} &= 4 \\ \frac{27}{5}-y &= \frac{20}{5} \\ -y &= -\frac{7}{5} \\ y &= \frac{7}{5} \end{aligned}$$

Section 1: Activity 5 (continued)

Check

LS	RS
$3x-y+2z$	4
$=3\left(\frac{27}{5}\right)-\frac{7}{5}+2\left(-\frac{27}{5}\right)$	
$=\frac{81}{5}-\frac{7}{5}-\frac{54}{5}$	
$=\frac{20}{5}$	
$=4$	

LS = RS

LS	RS
$2x+2y-z$	19
$=2\left(\frac{27}{5}\right)+2\left(\frac{7}{5}\right)-\left(-\frac{27}{5}\right)$	
$=\frac{54}{5}+\frac{14}{5}+\frac{27}{5}$	
$=\frac{95}{5}$	
$=19$	

LS = RS

LS	RS
$4x-2y-3z$	35
$=4\left(\frac{27}{5}\right)-2\left(\frac{7}{5}\right)-3\left(-\frac{27}{5}\right)$	
$=\frac{108}{5}-\frac{14}{5}+\frac{81}{5}$	
$=\frac{175}{5}$	
$=35$	

LS = RS

The solution is $x=\frac{27}{5}$, $y=\frac{7}{5}$, and $z=-\frac{27}{5}$, or $\left(\frac{27}{5}, \frac{7}{5}, -\frac{27}{5}\right)$.

b. Textbook question 51 of "Applications and Problem Solving," p. 45

51. Let x be number of \$5 bills; let y be the number of \$10 bills; and let z be the number of \$20 bills.

$$\begin{array}{l} \therefore\ x+y+z=71 \\ \quad 5x+10y+20z=925 \\ \quad z=x+y-7 \end{array} \xrightarrow{\text{Rearrange}} \begin{array}{ll} x+y+z=71 & ① \\ 5x+10y+20z=925 & ② \\ x+y-z=7 & ③ \end{array}$$

$$\begin{array}{rl} x+y+z=71 & ① \\ x+y-z=7 & ③ \\ \hline 2x+2y \qquad =78 & ①+③ \\ x+y=39 & ④ \end{array}$$

$$\begin{array}{rl}
20x+20y+20z=1420 & ⑤: 20\times ① \\
5x+10y+20z=\ \ 925 & ② \\
\hline
15x+10y\qquad\ \ =\ \ 495 & ⑤-② \\
3x+2y=99 & ⑥
\end{array}$$

$$\begin{array}{rl}
3x+2y=99 & ⑥ \\
2x+2y=78 & ⑦: 2\times ④ \\
\hline
x\qquad =21 & ⑥-⑦
\end{array}$$

Substitute $x=21$ into ④.

$$\begin{aligned}
x+y&=39\\
21+y&=39\\
y&=18
\end{aligned}$$

Substitute $x=21$ and $y=18$ into ①.

$$\begin{aligned}
x+y+z&=71\\
21+18+z&=71\\
z&=32
\end{aligned}$$

Check

LS	RS
$x+y+z$	71
$=21+18+32$	
$=71$	

LS = RS

LS	RS
$5x+10y+20z$	925
$=5(21)+10(18)+20(32)$	
$=105+180+640$	
$=925$	

LS = RS

LS	RS
z	$x+y-7$
$=32$	$=21+18-7$
	$=32$

LS = RS

There are 21 five-dollar bills, 18 ten-dollar bills, and 32 twenty-dollar bills in the register.

Section 1: Activity 5 (continued)

4. a. Textbook questions 27, 28, 30, 32, and 34 of “Practice,” p. 44

27. $x+y+z=14$ ①
$x-2y=-4$ ②
$z=9$ ③

Substitute $z=9$ into ①.

$$\begin{aligned} x+y+z&=14\\ x+y+9&=14\\ x+y&=5 \quad ④ \end{aligned}$$

$$\begin{array}{rcrl} x+y&=&5 & ④\\ x-2y&=&-4 & ②\\ \hline 3y&=&9 & ④-②\\ y&=&3 & \end{array}$$

Substitute $y=3$ into ④.

$$\begin{aligned} x+y&=5\\ x+3&=5\\ x&=2 \end{aligned}$$

Check

LS	RS
$x+y+z$	14
$=2+3+9$	
$=14$	

LS = RS

LS	RS
$x-2y$	-4
$=2-2(3)$	
$=2-6$	
$=-4$	

LS = RS

The solution is $x=2$, $y=3$, and $z=9$, or $(2,3,9)$.

28. $a-2b-3c=0$ ①

$b=2$ ②

$a+c=0$ ③

Substitute $b=2$ into ①.

$$\begin{aligned} a-2b-3c&=0\\ a-2(2)-3c&=0\\ a-4-3c&=0\\ a-3c&=4 \end{aligned}$$ ④

$$\begin{array}{r} a+c=0\\ \underline{a-3c=4}\\ 4c=-4\\ c=-1 \end{array}$$

③, ④, ③ − ④

Substitute $c=-1$ into ③.

$$\begin{aligned} a+c&=0\\ a+(-1)&=0\\ a&=1 \end{aligned}$$

Check

LS	RS
$a-2b-3c$	0
$=1-2(2)-3(-1)$	
$=1-4+3$	
$=0$	

LS = RS

LS	RS
$a+c$	0
$=1+(-1)$	
$=0$	

LS = RS

The solution is $a=1$, $b=2$, and $c=-1$, or $(1,2,-1)$.

Section 1: Activity 5 (continued)

30. $r+s=4$ ①

$2q+4r-s=-3$ ②

$3r=-3$ ③

Simplify ③.

$$\begin{aligned} 3r&=-3 \\ r&=-1 \end{aligned}$$

Substitute $r=-1$ into ①.

$$\begin{aligned} r+s&=4 \\ -1+s&=4 \\ s&=5 \end{aligned}$$

Substitute $r=-1$ and $s=5$ into ②.

$$\begin{aligned} 2q+4r-s&=-3 \\ 2q+4(-1)-5&=-3 \\ 2q-9&=-3 \\ 2q&=6 \\ q&=3 \end{aligned}$$

Check

LS	RS
$r+s$	4
$=-1+5$	
$=4$	

LS = RS

LS	RS
$2q+4r-s$	-3
$=2(3)+4(-1)-5$	
$=6-4-5$	
$=-3$	

LS = RS

LS	RS
$3r$	-3
$=3(-1)$	
$=-3$	

LS = RS

The solution is $q=3$, $r=-1$, and $s=5$, or $(3,-1,5)$.

32. $a+2b=-3$ ①

$4a-c=-10$ ②

$3c-2b=-6$ ③

Solve for a in ①.

$$\begin{aligned} a+2b&=-3\\ a&=-2b-3 \end{aligned}$$

Substitute $-2b-3$ for a into ②.

$$\begin{aligned} 4a-c&=-10\\ 4(-2b-3)-c&=-10\\ -8b-12-c&=-10\\ -8b-c&=2 \end{aligned}$$ ④

$-2b+3c=-6$ ③ ← Rearrange.

$-24b-3c=6$ ⑤: 3× ④

$-26b=0$ ③ + ⑤

$b=0$

Substitute $b=0$ into ① and ③.

$$\begin{aligned} a+2b&=-3\\ a+2(0)&=-3\\ a&=-3 \end{aligned}$$

$$\begin{aligned} 3c-2b&=-6\\ 3c-2(0)&=-6\\ 3c&=-6\\ c&=-2 \end{aligned}$$

Check

LS	RS
$a+2b$	-3
$=-3+2(0)$	
$=-3$	

LS = RS

LS	RS
$4a-c$	-10
$=4(-3)-(-2)$	
$=-12+2$	
$=-10$	

LS = RS

LS	RS
$3c-2b$	-6
$=3(-2)-2(0)$	
$=-6$	

LS = RS

The solution is $a=-3$, $b=0$, and $c=-2$, or $(-3,0,-2)$.

Section 1: Activity 5 (continued)

34. $5x+7y=-1$ ①

$-2y+3z=9$ ②

$7x-z=27$ ③

Solve for z in ③.

$$\begin{aligned} 7x-z&=27\\ z&=7x-27 \end{aligned}$$

Substitute $7x-27$ for z into ②.

$$\begin{aligned} -2y+3z&=9\\ -2y+3(7x-27)&=9\\ -2y+21x-81&=9\\ 21x-2y&=90 \quad ④ \end{aligned}$$

$$\begin{array}{rcl} 10x+14y&=&-2 \qquad ⑤ : 2\times ①\\ 147x-14y&=&630 \qquad ⑥ : 7\times ④\\ \hline 157x&=&628 \qquad ⑤+⑥\\ x&=&4 \end{array}$$

Substitute $x=4$ into ① and ③.

$$\begin{aligned} 5x+7y&=-1\\ 5(4)+7y&=-1\\ 20+7y&=-1\\ 7y&=-21\\ y&=-3 \end{aligned}$$

$$\begin{aligned} 7x-z&=27\\ 7(4)-z&=27\\ 28-z&=27\\ -z&=-1\\ z&=1 \end{aligned}$$

Check

LS	RS
$5x+7y$	-1
$=5(4)+7(-3)$	
$=20-21$	
$=-1$	

LS = RS

LS	RS
$-2y+3z$	9
$=-2(-3)+3(1)$	
$=6+3$	
$=9$	

LS = RS

LS	RS
$7x-z$	27
$=7(4)-1$	
$=28-1$	
$=27$	

LS = RS

The solution is $x=4$, $y=-3$, and $z=1$, or $(4,-3,1)$.

b. Textbook questions 41, 42, 43, and 49 of "Applications and Problem Solving," p. 45

41. a. For $f+s+t=159$, the total points earned by the first, second, and third-place winners was 159 points.

For $s-t=6$, the points earned by the second-place winner earned 6 more points than the third-place winner.

For $f=2s-3$, the points earned by the first-place winner equals three less than double the points earned by the second-place winner.

b.
$f+s+t=159$ ①
$s-t=6$ ②
$f=2s-3$ ③

Substitute $2s-3$ for f into ①.

$$\begin{aligned} f+s+t&=159 \\ (2s-3)+s+t&=159 \\ 3s+t&=162 \quad ④ \end{aligned}$$

$$\begin{array}{rl} s-t=\ \ 6 & ② \\ \underline{3s+t=162} & ④ \\ 4s\quad\ =168 & ②+④ \\ s=42 & \end{array}$$

Section 1: Activity 5 (continued)

Substitute $s = 42$ into ② and ③.

$$\begin{aligned} s - t &= 6 \\ 42 - t &= 6 \\ t &= 36 \end{aligned} \qquad \begin{aligned} f &= 2s - 3 \\ &= 2(42) - 3 \\ &= 84 - 3 \\ &= 81 \end{aligned}$$

Check

LS	RS
$f + s + t$	159
$= 81 + 42 + 36$	
$= 159$	

LS = RS

LS	RS
$s - t$	6
$= 42 - 36$	
$= 6$	

LS = RS

LS	RS
f	$2s - 3$
$= 81$	$= 2(42) - 3$
	$= 84 - 3$
	$= 81$

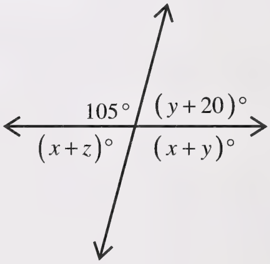
LS = RS

The first-place winner had 81 points; the second-place winner had 42 points; and the third-place winner had 36 points.

42. $x + y = 105$ ① ← Opposite angles are equal.

$x + z + 105 = 180$ ② ← Supplementary angles equal 180°.

$105 + y + 20 = 180$ ③ ← Supplementary angles equal 180°.



Simplify each equation.

$x + y = 105$ ①

$x + z = 75$ ②

$y = 55$ ③

Substitute $y = 55$ into ①.

$$\begin{aligned} x + y &= 105 \\ x + 55 &= 105 \\ x &= 50 \end{aligned}$$

Substitute $x=50$ into ②.

$$\begin{aligned} x+z&=75 \\ 50+z&=75 \\ z&=25 \end{aligned}$$

Check

LS	RS
$x+y$	105
$=50+55$	
$=105$	

LS = RS

LS	RS
$x+z+105$	180
$=50+25+105$	
$=180$	

LS = RS

LS	RS
$105+y+20$	180
$=105+55+20$	
$=180$	

LS = RS

The solution is $x=50$, $y=55$, and $z=25$, or $(50,55,25)$.

43. Let x, y, and z be the three numbers.

$$\begin{aligned} \therefore\ x+y&=22 \quad ① \\ y+z&=39 \quad ② \\ x+z&=45 \quad ③ \end{aligned}$$

Solve for y in ②.

$$\begin{aligned} y+z&=39 \\ y&=39-z \end{aligned}$$

Substitute $39-z$ for y into ①.

$$\begin{aligned} x+y&=22 \\ x+(39-z)&=22 \\ x-z&=-17 \quad ④ \end{aligned}$$

$$\begin{array}{rcrl} x+z & = & 45 & ③ \\ x-z & = & -17 & ④ \\ \hline 2x & = & 28 & ③+④ \\ x & = & 14 & \end{array}$$

Section 1: Activity 5 (continued)

Substitute $x = 14$ into ① and ③.

$$\begin{aligned} x + y &= 22 \\ 14 + y &= 22 \\ y &= 8 \end{aligned} \qquad\qquad \begin{aligned} x + z &= 45 \\ 14 + z &= 45 \\ z &= 31 \end{aligned}$$

Check

LS	RS
$x + y$	22
$= 14 + 8$	
$= 22$	

LS = RS

LS	RS
$y + z$	39
$= 8 + 31$	
$= 39$	

LS = RS

LS	RS
$x + z$	45
$= 14 + 31$	
$= 45$	

LS = RS

The numbers are 14, 8, and 31.

49. Let x be the time spent in Canada; let y be the time spent in England; and let z be the time spent in Africa.

$$\begin{aligned} \therefore\ x + y + z &= 61 \quad ① \\ x &= 3.5y \quad ② \\ y &= z + 5 \quad ③ \end{aligned}$$

Substitute $3.5y$ for x into ①.

$$\begin{aligned} x + y + z &= 61 \\ 3.5y + y + z &= 61 \\ 4.5y + z &= 61 \quad ④ \end{aligned}$$

$$\begin{array}{rl} 4.5y + z = 61 & ④ \\ \underline{\quad y - z = \ 5} & ③ \xleftarrow{\quad} \text{Rearrange.} \\ 5.5y \quad\ = 66 & ④ + ③ \\ y = 12 & \end{array}$$

Substitute $y = 12$ into ② and ③.

$$\begin{aligned} x &= 3.5y \\ &= 3.5(12) \\ &= 42 \end{aligned}$$

$$\begin{aligned} y &= z + 5 \\ 12 &= z + 5 \\ z &= 7 \end{aligned}$$

Check

LS	RS
$x + y + z$	61
$= 42 + 12 + 7$	
$= 61$	

LS = RS

LS	RS
x	$3.5y$
$= 42$	$= 3.5(12)$
	$= 42$

LS = RS

LS	RS
y	$z + 5$
$= 12$	$= 7 + 5$
	$= 12$

LS = RS

Margaret Laurence lived in Canada for 42 years.

5. a. Textbook questions 36 and 38 of "Practice," p. 44

36. $x + 0.5y + z = -8$ ①

$0.2x - y + 1.5z = -4.6$ ②

$0.05x + 0.02y - 0.1z = 0.21$ ③

Simplify each equation.

$2x + y + 2z = -16$ ① ◄— Multiply both sides by 2.

$2x - 10y + 15z = -46$ ② ◄— Multiply both sides by 10.

$5x + 2y - 10z = 21$ ③ ◄— Multiply both sides by 100.

$$\begin{aligned} 2x + y + 2z &= -16 && ① \\ 2x - 10y + 15z &= -46 && ② \\ \hline 11y - 13z &= 30 && ④: ① - ② \end{aligned}$$

$$\begin{aligned} 10x + 5y + 10z &= -80 && ⑤: 5 \times ① \\ 10x + 4y - 20z &= 42 && ⑥: 2 \times ③ \\ \hline y + 30z &= -122 && ⑦: ⑤ - ⑥ \end{aligned}$$

$$\begin{aligned} 11y - 13z &= 30 && ④ \\ 11y + 330z &= -1342 && ⑧: 11 \times ⑦ \\ \hline -343z &= 1372 \\ z &= -4 \end{aligned}$$

Section 1: Activity 5 (continued)

Substitute $z=-4$ into ⑦.

$$\begin{aligned} y+30z&=-122 \\ y+30(-4)&=-122 \\ y-120&=-122 \\ y&=-2 \end{aligned}$$

Substitute $y=-2$ and $z=-4$ into ①.

$$\begin{aligned} 2x+y+2z&=-16 \\ 2x+(-2)+2(-4)&=-16 \\ 2x-2-8&=-16 \\ 2x&=-6 \\ x&=-3 \end{aligned}$$

Check

LS	RS
$x+0.5y+z$	-8
$=-3+0.5(-2)+(-4)$	
$=-3-1-4$	
$=-8$	

LS = RS

LS	RS
$0.2x-y+1.5z$	-4.6
$=0.2(-3)-(-2)+1.5(-4)$	
$=-0.6+2-6$	
$=-4.6$	

LS = RS

LS	RS
$0.05x+0.02y-0.1z$	0.21
$=0.05(-3)+0.02(-2)-0.1(-4)$	
$=-0.15-0.04+0.4$	
$=0.21$	

LS = RS

The solution is $x=-3$, $y=-2$, and $z=-4$, or $(-3,-2,-4)$.

38. $\frac{r}{2}-\frac{s}{3}+\frac{t}{6}=2$ ①

$\frac{r}{4}+\frac{s}{2}-\frac{t}{3}=-8$ ②

$\frac{r}{2}+\frac{s}{3}+\frac{t}{4}=-1$ ③

Simplify each equation.

$3r-2s+t=12$ ① ← Multiply both sides by 6, the LCD.

$3r+6s-4t=-96$ ② ← Multiply both sides by 12, the LCD.

$6r+4s+3t=-12$ ③ ← Multiply both sides by 12, the LCD.

$$\begin{array}{rl} 3r-2s+t= & 12 \quad \textcircled{1}\\ 3r+6s-4t= & -96 \quad \textcircled{2}\\ \hline -8s+5t= & 108 \quad \textcircled{4}:\textcircled{1}-\textcircled{2} \end{array} \qquad \begin{array}{rl} 6r-4s+2t= & 24 \quad \textcircled{5}:2\times\textcircled{1}\\ 6r+4s+3t= & -12 \quad \textcircled{3}\\ \hline -8s-t= & 36 \quad \textcircled{6}:\textcircled{5}-\textcircled{3} \end{array}$$

$$\begin{array}{rl} -8s+5t= & 108 \quad \textcircled{4}\\ -8s-t= & 36 \quad \textcircled{6}\\ \hline 6t= & 72 \quad \textcircled{4}-\textcircled{6}\\ t= & 12 \end{array}$$

Substitute $t=12$ into ⑥.

$$\begin{aligned} -8s-t&=36\\ -8s-12&=36\\ -8s&=48\\ s&=-6 \end{aligned}$$

Substitute $s=-6$ and $t=12$ into ①.

$$\begin{aligned} 3r-2s+t&=12\\ 3r-2(-6)+12&=12\\ 3r+12+12&=12\\ 3r&=-12\\ r&=-4 \end{aligned}$$

Section 1: Activity 5 (continued)

Check

LS	RS
$\frac{r}{2}-\frac{s}{3}+\frac{t}{6}$	2
$=\frac{-4}{2}-\left(\frac{-6}{3}\right)+\frac{12}{6}$	
$=-2+2+2$	
$=2$	

LS = RS

LS	RS
$\frac{r}{4}+\frac{s}{2}-\frac{t}{3}$	-8
$=\frac{-4}{4}+\left(\frac{-6}{2}\right)-\frac{12}{3}$	
$=-1-3-4$	
$=-8$	

LS = RS

LS	RS
$\frac{r}{2}+\frac{s}{3}+\frac{t}{4}$	-1
$=\frac{-4}{2}+\left(\frac{-6}{3}\right)+\frac{12}{4}$	
$=-2-2+3$	
$=-1$	

LS = RS

The solution is $r=-4$, $s=-6$, and $t=12$, or $(-4,-6,12)$.

b. **Textbook questions 52, 53, and 57 of "Applications and Problem Solving," pp. 45 and 46**

52. Let x be the length of Canada's coastline; let y be the length of the U.S.'s coastline; and let z be the length of Mexico's coastline.

$\therefore\ x+y+z=270\ 000$ ①

$x=16\left(\frac{y+z}{2}\right)$ ②

$y=z+10\ 000$ ③

Simplify ②.

$$\begin{aligned} x &= \overset{8}{\cancel{16}}\left(\frac{y+z}{\underset{1}{\cancel{2}}}\right) \\ &= 8y+8z \end{aligned}$$

Substitute $8y+8z$ for x into ①.

$$\begin{aligned} x+y+z &= 270\,000 \\ (8y+8z)+y+z &= 270\,000 \\ 9y+9z &= 270\,000 \\ y+z &= 30\,000 \quad ④ \end{aligned}$$

$$\begin{array}{rll} y-z = 10\,000 & ③ & \longleftarrow \text{Rearrange.} \\ y+z = 30\,000 & ④ & \\ \hline 2y \qquad = 40\,000 & ③+④ & \\ y = 20\,000 & & \end{array}$$

Substitute $y = 20\,000$ into ③.

$$\begin{aligned} y &= z+10\,000 \\ 20\,000 &= z+10\,000 \\ z &= 10\,000 \end{aligned}$$

Substitute $y = 20\,000$ and $z = 10\,000$ into ①.

$$\begin{aligned} x+y+z &= 270\,000 \\ x+20\,000+10\,000 &= 270\,000 \\ x+30\,000 &= 270\,000 \\ x &= 240\,000 \end{aligned}$$

Section 1: Activity 5 (continued)

Check

LS	RS
$x+y+z$	270 000
$=240\,000+20\,000+10\,000$	
$=270\,000$	

LS = RS

LS	RS
x	$16\left(\frac{y+z}{2}\right)$
$=240\,000$	$=16\left(\frac{20\,000+10\,000}{2}\right)$
	$=\overset{8}{\cancel{16}}\left(\frac{30\,000}{\underset{1}{\cancel{2}}}\right)$
	$=240\,000$

LS = RS

LS	RS
y	$z+10\,000$
$=20\,000$	$=10\,000+10\,000$
	$=20\,000$

LS = RS

The length of Canada's coastline is 240 000 km; the length of the U.S.'s coastline is 20 000 km; and the length of Mexico's coastline is 10 000 km.

53. Let x be the area of forested land in British Columbia; let y be the area of forested land in Alberta; and let z be the area of forested land in Manitoba.

$\therefore\ x+y+z=1330\,000$ ①

$y=z$ ②

$x=0.9(y+z)$ ③

Simplify ③.

$x=0.9y+0.9z$

$10x=9y+9z$ ④ ← Multiply both sides by 10.

Substitute z for y into ① and ④.

$x+y+z=1\,330\,000$

$x+z+z=1\,330\,000$

$x+2z=1\,330\,000$ ⑤

$10x=9y+9z$

$10x=9z+9z$

$10x-18z=0$

$5x-9z=0$ ⑥

$5x+10z=6\,650\,000$ ⑦: $5\times$ ⑤

$5x-\ \ 9z=\qquad 0$ ⑥

$19z=6\,650\,000$ ⑦ − ⑥

$z=350\,000$

$\therefore\ y=350\,000$

Substitute $z=350\,000$ into ⑤.

$$\begin{aligned} x+2z&=1\,330\,000\\ x+2(350\,000)&=1\,330\,000\\ x+700\,000&=1\,330\,000\\ x&=630\,000 \end{aligned}$$

Check

LS	RS
$x+y+z$	$1\ 330\ 000$
$=630\,000+350\,000+350\,000$	
$=1\,330\,000$	

LS = RS

LS	RS
y	z
$=350\,000$	$=350\,000$

LS = RS

LS	RS
x	$0.9(y+z)$
$=630\,000$	$=0.9(350\,000+350\,000)$
	$=0.9(700\,000)$
	$=630\,000$

LS = RS

British Columbia has 630 000 km^2 of forested land; Alberta has 350 000 km^2 of forested land; and Manitoba has 350 000 km^2 of forested land.

Section 1: Activity 5 (continued)

57. Let x be the mass of alloy X; let y be the mass of alloy Y; and let z be the mass of alloy Z.

$$\begin{aligned} \therefore\ 0.5x+0.6y+0.7z&=57 && \text{①}\\ 0.4x+0.2y+0.1z&=28 && \text{②}\\ 0.1x+0.2y+0.2z&=15 && \text{③} \end{aligned}$$

Simplify by multiplying each equation by 10.

$$\begin{aligned} 5x+6y+7z&=570 && \text{①}\\ 4x+2y+z&=280 && \text{②}\\ x+2y+2z&=150 && \text{③} \end{aligned}$$

$$\begin{array}{rl} 5x+6y+7z=570 & \text{①}\\ 12x+6y+3z=840 & \text{④} : 3\times\text{②}\\ \hline -7x\qquad +4z=-270 & \text{⑤} : \text{①}-\text{④} \end{array}$$

$$\begin{array}{rl} 4x+2y+z=280 & \text{②}\\ x+2y+2z=150 & \text{③}\\ \hline 3x\qquad -z=130 & \text{⑥} : \text{②}-\text{③} \end{array}$$

$$\begin{array}{rl} -7x+4z=-270 & \text{⑤}\\ 12x-4z=520 & \text{⑦} : 4\times\text{⑥}\\ \hline 5x\qquad =250 & \text{⑤}+\text{⑦}\\ x=50 & \end{array}$$

Substitute $x=50$ into ⑥.

$$\begin{aligned} 3x-z&=130\\ 3(50)-z&=130\\ 150-z&=130\\ z&=20 \end{aligned}$$

Substitute $x=50$ and $z=20$ into ③.

$$\begin{aligned} x+2y+2z&=150\\ 50+2y+2(20)&=150\\ 50+2y+40&=150\\ 2y&=60\\ y&=30 \end{aligned}$$

Check

LS	RS
$0.5x+0.6y+0.7z$	57
$=0.5(50)+0.6(30)+0.7(20)$	
$=25+18+14$	
$=57$	

LS = RS

LS	RS
$0.4x+0.2y+0.1z$	28
$=0.4(50)+0.2(30)+0.1(20)$	
$=20+6+2$	
$=28$	

LS = RS

LS	RS
$0.1x+0.2y+0.2z$	15
$=0.1(50)+0.2(30)+0.2(20)$	
$=5+6+4$	
$=15$	

LS = RS

To make 100 g of the new alloy, 50 g of alloy X, 30 g of alloy Y, and 20 g of alloy Z are needed.

6. a. Textbook questions 1 to 3 of Investigation 1, "Interpreting the Graph," p. 50

1. The graphs intersect at approximately $(14, 50)$; therefore, the populations were the same about 14 million years ago.

2. Population percentages for today are read at $t=0$.

- **a.** Apes make up approximately 6% of the population.
- **b.** Monkeys make up approximately 94% of the population.

3. **a.** Apes made up approximately 94% of the population.
- **b.** Monkeys made up approximately 6% of the population.

Section 1: Activity 5 (continued)

b. Textbook questions 1 and 2 of Investigation 2, "Solving Algebraically," p. 51

1. **Equation 1: Apes**

Find the slope; then use the point-slope formula. Let $(x_1, y_1) = (20, 80)$ and $(x_2, y_2) = (10, 30)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{30 - 80}{10 - 20} \\ &= \frac{-50}{-10} \\ &= 5 \end{aligned}$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 80 &= 5(x - 20) \\ y - 80 &= 5x - 100 \\ 5x - y &= 20 \end{aligned}$$

Equation 2: Monkeys

Find the slope; then use the point-slope formula. Let $(x_1, y_1) = (20, 20)$ and $(x_2, y_2) = (10, 70)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{70 - 20}{10 - 20} \\ &= \frac{50}{-10} \\ &= -5 \end{aligned}$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 20 &= -5(x - 20) \\ y - 20 &= -5x + 100 \\ 5x + y &= 120 \end{aligned}$$

2. **a.**

$$\begin{array}{rl} 5x - y = 20 & ① \\ 5x + y = 120 & ② \\ \hline 10x \quad = 140 & ① + ② \\ x = 14 & \end{array}$$

Substitute $x = 14$ into ①.

$$\begin{aligned} 5x - y &= 20 \\ 5(14) - y &= 20 \\ 70 - y &= 20 \\ -y &= -50 \\ y &= 50 \end{aligned}$$

Check

LS	RS
$5x - y$	20
$= 5(14) - 50$	
$= 70 - 50$	
$= 20$	

LS = RS

LS	RS
$5x + y$	120
$= 5(14) + 50$	
$= 70 + 50$	
$= 120$	

LS = RS

The point of intersection is $(14, 50)$.

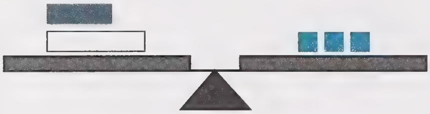
b. The coordinates of the intersection show that the population of monkeys and apes each constituted 50% of the population 14 million years ago.

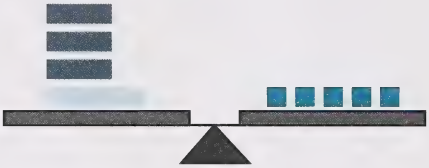
Section 1: Follow-up Activities

Extra Help

1. **Textbook question 2 of Investigation 2, "Representing Systems of Equations," p. 18**

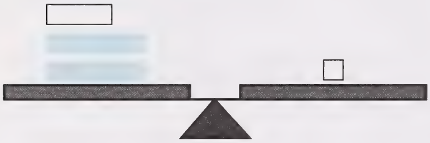
2. $y-x=3$ ① $3y+x=5$ ②

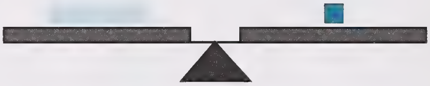




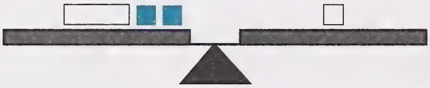
2. **Textbook question 2 of Investigation 3, "Solving Systems by Substitution, I," p. 19**

2. $2x-y=-1$ ① $x=1$ ②



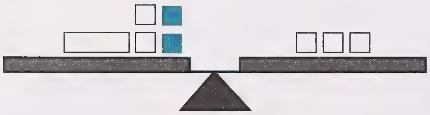


Substitute a 1-tile for every *x*-tile on the balance scale for ①.



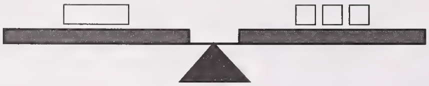
$$2(1)-y=-1$$
$$2-y=-1$$

Isolate the variable *y* by adding two −1-tiles to both sides of the balance scale.



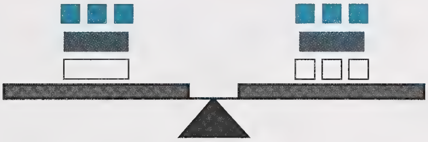
$$2-y+(-2)=-1+(-2)$$
$$2-y-2=-1-2$$

Simplify by removing the zero pairs.



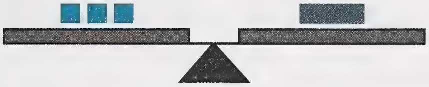
$$-y=-3$$

Add one y-tile to both sides to have the variable represented in a positive manner and then add three 1-tiles to both sides to isolate the variable.



$$-y+y+3=-3+y+3$$

Simplify by removing the zero pairs.



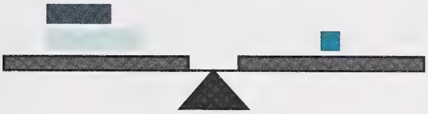
$$3=y$$

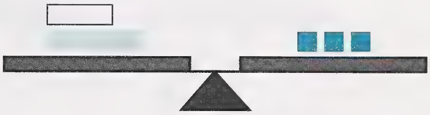
Therefore, the solution is $(1,3)$.

3. **Textbook question 2 of Investigation 4, "Solving Systems by Substitution, II," p. 20**

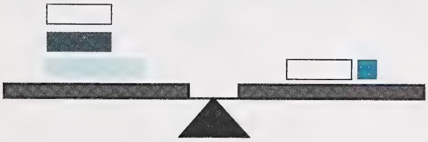
2. $x+y=1$ ①

$x-y=3$ ②



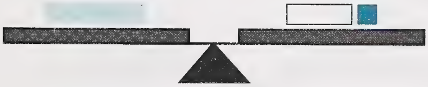


Isolate x in ① by adding one $-y$-tile to both sides.



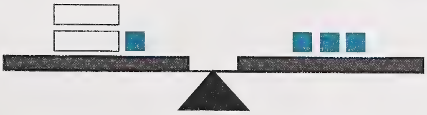
$$x+y+(-y)=1+(-y)$$
$$x+y-y=1-y$$

Simplify by removing the zero pair.



$$x=1-y$$

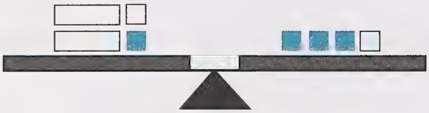
Substitute $-y+1$ for x into ② using algebra tiles.



$$(1-y)-y=3$$
$$1-y-y=3$$

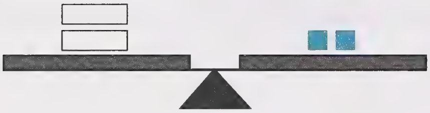
Extra Help (continued)

Add one −1-tile to both sides.



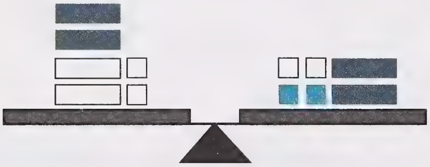
$$1-2y+(-1)=3+(-1)$$
$$1-2y-1=3-1$$

Simplify by removing zero pairs.



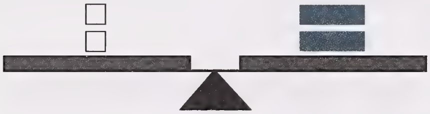
$$-2y=2$$

To isolate the variable *y*, add two *y*-tiles and two −1-tiles to both sides.



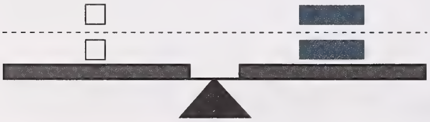
$$-2y+2y+2(-1)=2+2y+2(-1)$$
$$-2y+2y-2=2+2y-2$$

Simplify by removing the zero pairs.



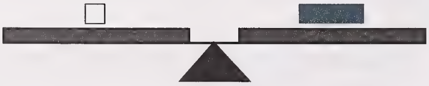
$$-2=2y$$

Group tiles to show the value of the variable.



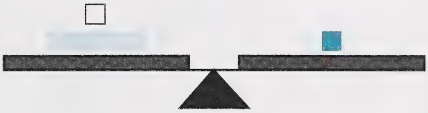
$$\frac{-2}{2}=\frac{2y}{2}$$

Show the result.



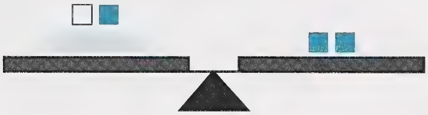
$$-1=y$$

Substitute -1 for y into ① using algebra tiles.



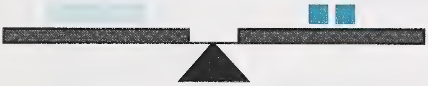
$$x+(-1)=1$$
$$x-1=1$$

Add one 1-tile to both sides.



$$x-1+1=1+1$$

Simplify by removing the zero pair.

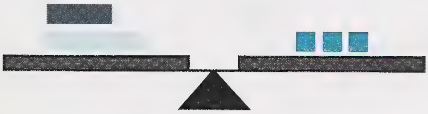


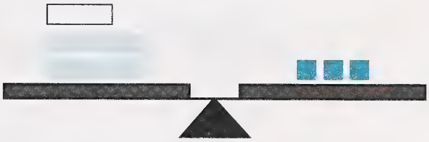
$$x=2$$

The solution is $(2,-1)$.

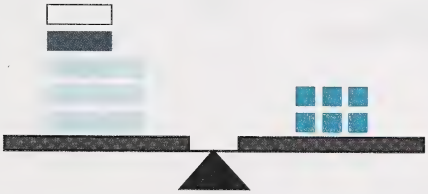
4. a. Textbook question 1 of Investigation 1, "Solving Systems by Addition," p. 32

1. $x+y=3$ ① $2x-y=3$ ②





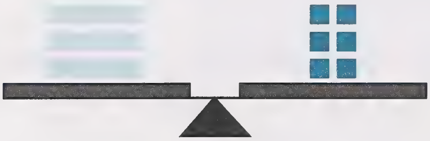
Add the equations to eliminate y.



$$(x+y)+(2x-y)=3+3$$
$$x+y+2x-y=3+3$$

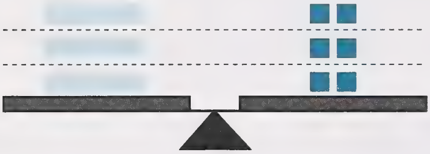
Extra Help (continued)

Simplify by removing the zero pair.



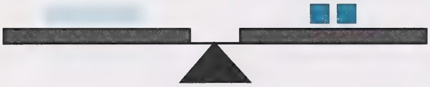
$$3x = 6$$

Group tiles to show the value of the variable.



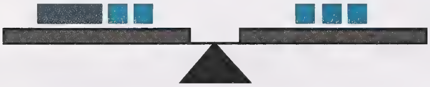
$$\frac{3x}{3} = \frac{6}{3}$$

Show the result.



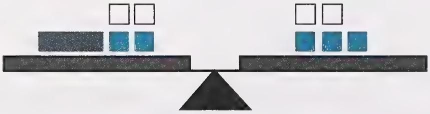
$$x = 2$$

Substitute 2 for x into ① using algebra tiles.



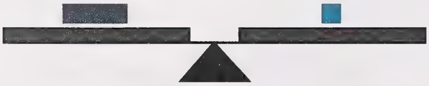
$$2 + y = 3$$

Add two -1-tiles to both sides.



$$2 + y + 2(-1) = 3 + 2(-1)$$
$$2 + y - 2 = 3 - 2$$

Simplify by removing the zero pairs.

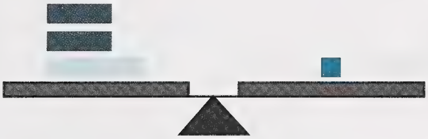


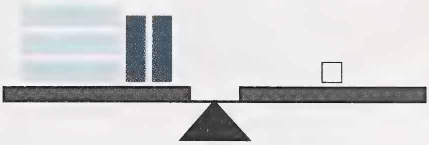
$$y = 1$$

The solution is $(2, 1)$.

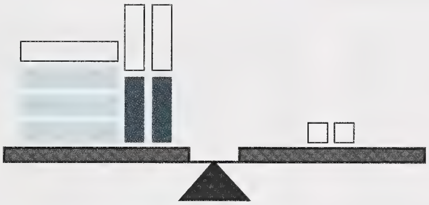
b. Textbook question 2 of Investigation 2, "Solving Systems by Subtraction," p. 33

2. $x + 2y = 1$ ①　　$3x + 2y = -1$ ②



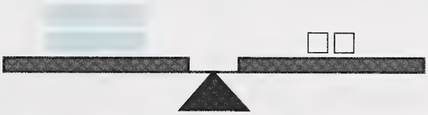


Because the coefficient of y in the two equations is the same, subtract the equations to eliminate y. Add the opposite of ① to ②.



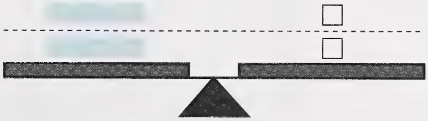
$$(3x + 2y) + (-x - 2y) = -1 + (-1)$$
$$3x + 2y - x - 2y = -1 - 1$$

Simplify by removing the zero pairs.



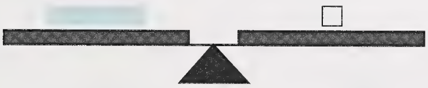
$$2x = -2$$

Group tiles to show the value of the variable.



$$\frac{2x}{2} = \frac{-2}{2}$$

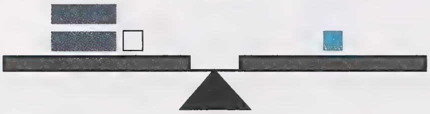
Show the result.



$$x = -1$$

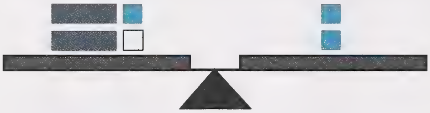
Extra Help (continued)

Substitute -1 for x into ① using algebra tiles.



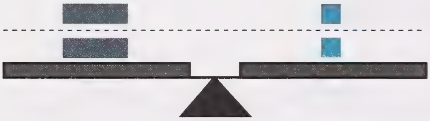
$$-1+2y=1$$

Add one 1-tile to both sides.



$$-1+2y+1=1+1$$

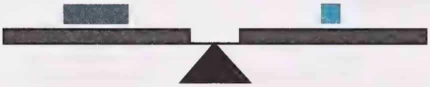
Simplify by removing the zero pair; then group the tiles to show the value of the variable.



$$2y=2$$

$$\frac{2y}{2}=\frac{2}{2}$$

Show the result.



$$y=1$$

The solution is $(-1,1)$.

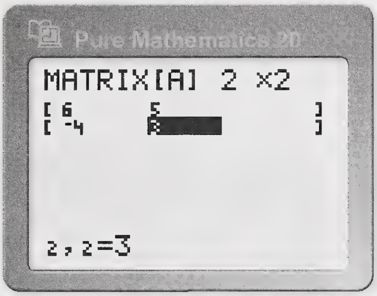
Enrichment

1. Textbook questions 18, 35, and 36 of "Practice," p. 39

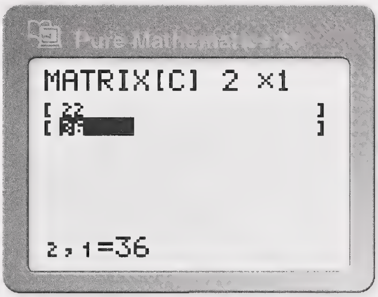
18. $6x + 5y = 22$ ①
$3y = 4x + 36$ ②

Rearrange the system of equations.

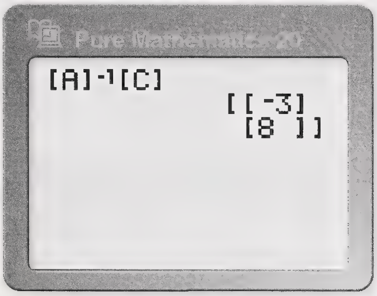
$6x + 5y = 22$ ①
$-4x + 3y = 36$ ②



```
MATRIX[A] 2 ×2
[ 6      5      ]
[ -4     3      ]

2,2=3
```



```
MATRIX[C] 2 ×1
[ 22            ]
[ 36            ]

2,1=36
```

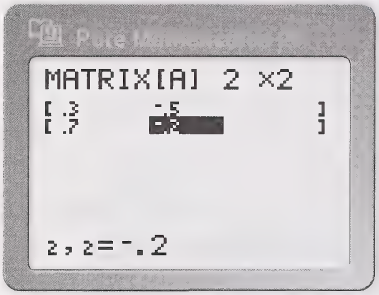


```
[A]⁻¹[C]
              [[-3]
               [8 ]]
```

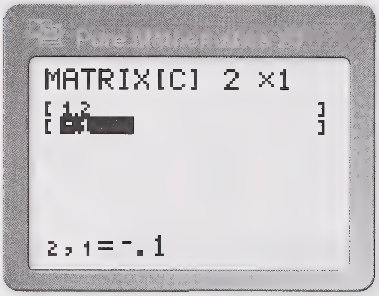
The solution is $(-3, 8)$.

Enrichment (continued)

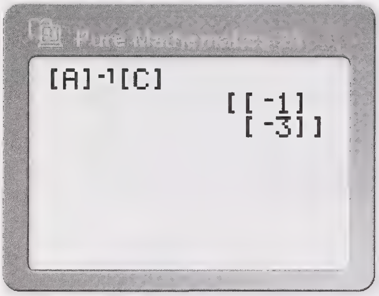
35. $0.3x - 0.5y = 1.2$ ①
$0.7x - 0.2y = -0.1$ ②



```
MATRIX[A] 2 ×2
[ .3      -.5       ]
[ .7      -.2       ]

2,2=-.2
```



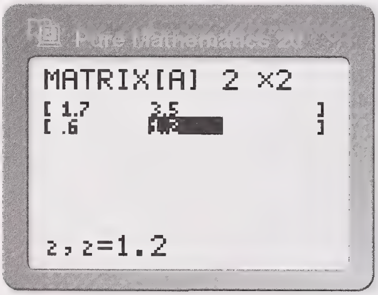
```
MATRIX[C] 2 ×1
[ 1.2               ]
[ -.1               ]

2,1=-.1
```



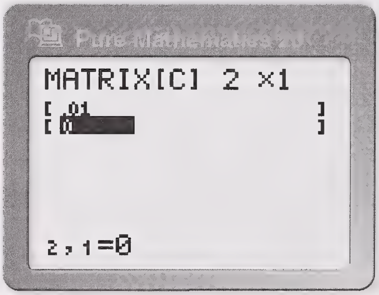
```
[A]-1[C]
                [[-1]
                 [-3]]
```

The solution is $(-1, -3)$.

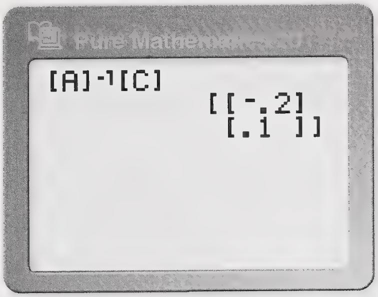
36. $1.7x + 3.5y = 0.01$ ①
$0.6x + 1.2y = 0$ ②



```
MATRIX[A] 2 ×2
[ 1.7     3.5       ]
[ .6      1.2       ]

2,2=1.2
```



```
MATRIX[C] 2 ×1
[ .01               ]
[ 0                 ]

2,1=0
```



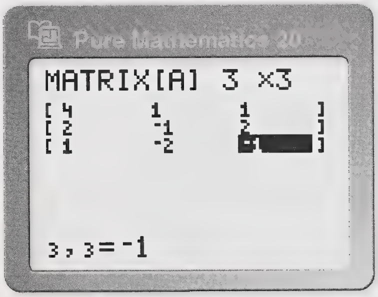
```
[A]⁻¹[C]
            [[-.2]
             [.1 ]]
```

The solution is $(-0.2, 0.1)$.

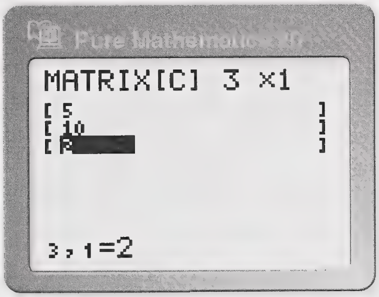
2. **a.** $4x + y + z = 5$ ①

$2x - y + 2z = 10$ ②

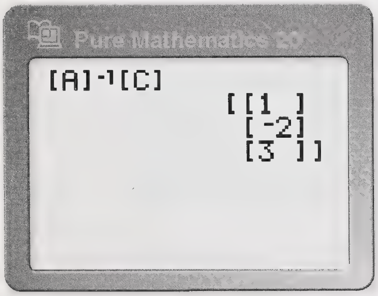
$x - 2y - z = 2$ ③



```
MATRIX[A] 3 ×3
[4     1     1    ]
[2     -1    2    ]
[1     -2    -1   ]

3,3=-1
```



```
MATRIX[C] 3 ×1
[5          ]
[10         ]
[2          ]

3,1=2
```



```
[A]⁻¹[C]
            [[1 ]
             [-2]
             [3 ]]
```

The solution is $x = 1$, $y = -2$, and $z = 3$, or $(1, -2, 3)$.

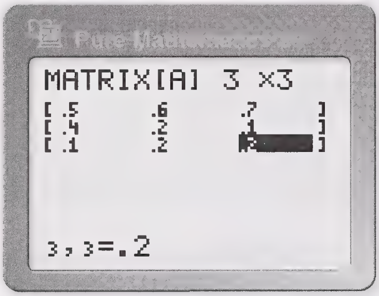
Enrichment (continued)

b. Textbook question 57 of "Applications and Problem Solving," p. 46

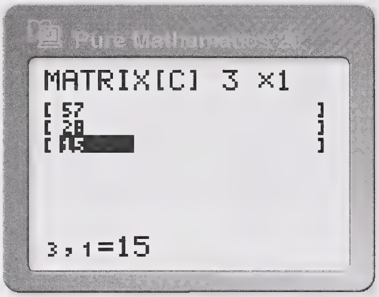
57. $0.5x+0.6y+0.7z=57$ ①

$0.4x+0.2y+0.1z=28$ ②

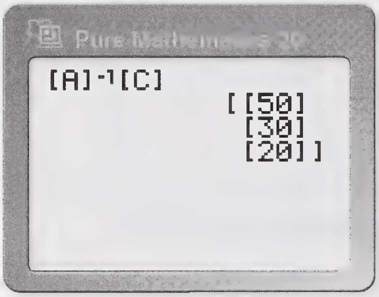
$0.1x+0.2y+0.2z=15$ ③



```
MATRIX[A] 3 ×3
[ .5      .6      .7    ]
[ .4      .2      .1    ]
[ .1      .2      .2    ]

3,3=.2
```



```
MATRIX[C] 3 ×1
[ 57           ]
[ 28           ]
[ 15           ]

3,1=15
```



```
[A]⁻¹[C]
               [[50]
                [30]
                [20]]
```

The solution is $x=50$, $y=30$, and $z=20$, or $(50,30,20)$.

To make 100 g of the new alloy, 50 g of alloy X, 30 g of alloy Y, and 20 g of alloy Z are needed.

Section 2: Activity 1

1. **Textbook question 1 of "Frequency Ranges," p. 58**

1. a. $250 \le f \le 21\,000$ **b.** $760 \le f \le 1520$

2. **Textbook questions 15, 16, 19, 20, and 24 of "Warm Up," p. 59**

15. $x>-4$ **16.** $x \le -2$ **19.** $-2 \le x < 4$ **20.** $-6 < x \le -2$

24. The inequalities $-3 \le x < 5$ and $5 > x \ge -3$ are equivalent. The same range is indicated by reversing the endpoints and the inequality signs. This shows that order does not matter.

3. a. Textbook questions a. and b. of "Explore: Solve the Inequalities," p. 60

a. $s-320=0$ $\quad s=320$ $\qquad s-320=80$ $\quad s=400$

b. $s-320\ge 0$ $\quad s\ge 320$ $\qquad s-320\le 80$ $\quad s\le 400$

b. Textbook questions 1, 2, 4.b., 4.d., 5, and 6 of "Inquire," pp. 60 and 61

1. The lowest speed at which a passenger aircraft can descend is 320 km/h.

2. The highest speed at which a passenger aircraft can descend is 400 km/h.

4. b.

$$7x+2>23$$
$$7x>21$$
$$x>3$$

d.

$$\frac{1}{2}x-5<3$$
$$\frac{1}{2}x<8$$
$$x<16$$

5. a.

Original Inequality	Operation	Resulting Inequality
$9>6$	Add 3	$9+3>6+3$
$9>6$	Subtract 3	$9-3>6-3$
$9>6$	Multiply by 3	$9\times 3>6\times 3$
$9>6$	Multiply by -3	$9\times(-3)<6\times(-3)$
$9>6$	Divide by 3	$\frac{9}{3}>\frac{6}{3}$
$9>6$	Divide by -3	$\frac{9}{-3}<\frac{6}{-3}$

b. Multiplying or dividing an inequality by a negative number reverses the direction of the inequality symbol.

6. a. $9>2$ **b.** $1<5$ **c.** $-5<-3$ **d.** $1>-2$

e. $8>-4$ **f.** $9>6$ **g.** $8>6$ **h.** $4>3$

i. $1<2$ **j.** $-1<1$ **k.** $-2<-1$ **l.** $1<2$

Section 2: Activity 1 (continued)

4. a. Textbook questions 12, 16, 26, 30, 34, and 36 of "Practice," p. 63

12.
$$\begin{aligned} 4z-3 &\ge 3z+2 \\ 4z &\ge 3z+5 \\ z &\ge 5 \end{aligned}$$

Check

Try $z=6$.

LS	RS
$4z-3$	$3z+2$
$=4(6)-3$	$=3(6)+2$
$=24-3$	$=18+2$
$=21$	$=20$

LS $\ge$ RS

The solution is $z \ge 5$.

16.
$$\begin{aligned} 4(2x+1) &\ge 2 \\ 8x+4 &\ge 2 \\ 8x &\ge -2 \\ x &\ge -\frac{1}{4} \end{aligned}$$

Check

Try $x=0$.

LS	RS
$4(2x+1)$	2
$=4[2(0)+1]$	
$=4(1)$	
$=4$	

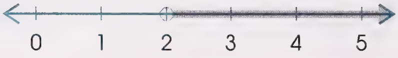
LS $\ge$ RS

The solution is $x \ge -\frac{1}{4}$.

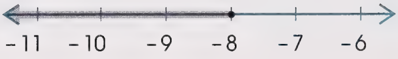
26.
$$\begin{aligned} 8-3x &< 5 \\ -3x &< -3 \\ \frac{-3x}{-3} &> \frac{-3}{-3} \\ x &> 1 \end{aligned}$$

30.
$$\begin{aligned} -2(3+x) &< 4(x-2) \\ -6-2x &< 4x-8 \\ -6x &< -2 \\ \frac{-6x}{-6} &> \frac{-2}{-6} \\ x &> \frac{1}{3} \end{aligned}$$

34.
$$\begin{aligned} \frac{w}{2}+2 &> 3 \\ \frac{w}{2} &> 1 \\ w &> 2 \end{aligned}$$



36.
$$\begin{aligned} \frac{3z}{4}+5 &\le -1 \\ \frac{3z}{4} &\le -6 \\ z &\le -8 \end{aligned}$$



b. Textbook questions 51 to 54, 56, and 60 of "Applications and Problem Solving," pp. 63 and 64

51. $$\begin{aligned} 4m+18 &\le 50 \\ 4m &\le 32 \\ m &\le 8 \end{aligned}$$

Katrina can buy up to 8 markers.

52. $$\begin{aligned} 12.2 &\ge 7d-0.4 \\ 12.6 &\ge 7d \\ 1.8 &\ge d \end{aligned}$$

The lion can leap up to 1.8 m.

53. $$\begin{aligned} 5x+10 &> 90 & \text{and} \quad 5x+10 &< 180 \\ 5x &> 80 & 5x &< 170 \\ x &> 16 & x &< 34 \end{aligned}$$

$\therefore\ 16 < x < 34$

54. **a.** Total cost $= 12.25 + 1.55n$

b. $$\begin{aligned} 12.25+1.55n &\le 20.00 \\ 1.55n &\le 7.75 \\ n &\le 5 \end{aligned}$$

You can afford 5 extra toppings.

56. Let x represent the number of caps.

Therefore, the total cost is $500 + 7x$ and the total income is $15x$.

In order to make a profit, total income must be greater than total cost.

$$\begin{aligned} \therefore\ 15x &> 500+7x \\ 8x &> 500 \\ x &> 62.5 \end{aligned}$$

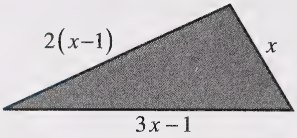
Because x is restricted to the natural numbers, $x \ge 63$.

Therefore, a minimum of 63 caps must be ordered in one batch in order to raise money.

Section 2: Activity 1 (continued)

60.

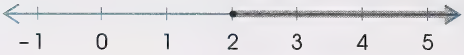
$$\begin{aligned}
12 &\le 2(x-1)+x+3x-1 \le 15\\
12 &\le 2x-2+x+3x-1 \le 15\\
12 &\le 6x-3 \le 15\\
15 &\le 6x \le 18\\
\frac{15}{6} &\le x \le \frac{18}{6}\\
\frac{5}{2} &\le x \le 3
\end{aligned}$$



5. Texbook questions 39, 45, 48, and 50 of "Practice," p. 63

39.

$$\begin{aligned}
1.9 &\ge 4.9-1.5q\\
-3 &\ge -1.5q\\
2 &\le q\\
q &\ge 2
\end{aligned}$$



45.

$$\begin{aligned}
1.5(x+2)+1 &> 2.5(1-x)-0.5\\
15(x+2)+10 &> 25(1-x)-5\\
15x+30+10 &> 25-25x-5\\
15x+40 &> 20-25x\\
40x+40 &> 20\\
40x &> -20\\
x &> -0.5
\end{aligned}$$

The contents of the brackets stay the same; only one factor of each term is multiplied by 10. In this case, the coefficients of the brackets increase by a factor of 10.

48.

$$\frac{2-x}{2} \ge \frac{2x+1}{4}$$

$$\overset{2}{\cancel{4}}\left(\frac{2-x}{\underset{1}{\cancel{2}}}\right) \ge \cancel{4}\left(\frac{2x+1}{\cancel{4}}\right)$$

← Multiply both sides by 4, the LCD.

$$\begin{aligned}
4-2x &\ge 2x+1\\
-2x &\ge 2x-3\\
-4x &\ge -3\\
\frac{-4x}{-4} &\le \frac{-3}{-4}\\
x &\le \frac{3}{4}
\end{aligned}$$

50. $$\frac{2-3x}{2}+\frac{2}{3}\le\frac{3x-2}{6}$$

$$\overset{3}{\cancel{6}}\left(\frac{2-3x}{\underset{1}{\cancel{2}}}\right)+\overset{2}{\cancel{6}}\left(\frac{2}{\underset{1}{\cancel{3}}}\right)\le\cancel{6}\left(\frac{3x-2}{\cancel{6}}\right)$$ ← Multiply both sides by 6, the LCD.

$$6-9x+4\le3x-2$$

$$10-9x\le3x-2$$

$$-9x\le3x-12$$

$$-12x\le-12$$

$$\frac{-12x}{-12}\ge\frac{-12}{-12}$$

$$x\ge1$$

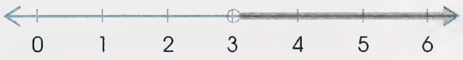
6. a. Textbook questions 1, 3, and 4 of Investigation 1, "Displaying a Solution," p. 65

1. $2x+1<3x-2$

$1<x-2$

$3<x$

$x>3$



3. The boundary point $x=3$ is not included because the Trace feature shows that $y=0$ for this value of x. **Remember:** Press $\boxed{\text{ZOOM}}$ $\boxed{4}$ to see the graph more clearly.

4. a. The ZOOM instruction allows ZDecimal to be used. This allows you to trace using exact decimal values of x. The Trace instruction allows you to examine the boundary point.

b. The TABLE menu allows a range of values to be viewed at once. The TABLE SETUP menu allows values extremely close to a critical value to be examined. Simply adjust ΔTbl.

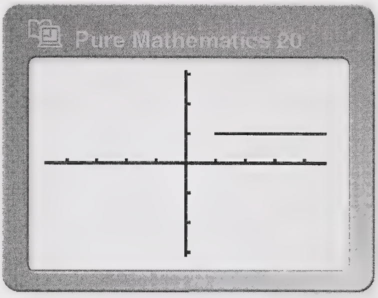
c. By changing the viewing window, you may be able to see the endpoint of an interval that was originally off the display.

Section 2: Activity 1 (continued)

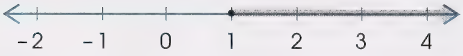
b. Textbook questions 5, 9, and 13 of Investigation 2, “Solving Inequalities,” p. 65

5. $-4x+2\leq-2$

Y= (–) 4 X,T,Θ,*n* + 2 2nd [TEST]
6 (–) 2 GRAPH



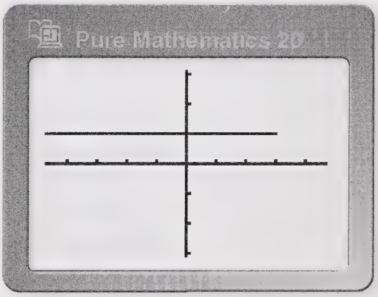
The graph should show that the boundary point and all values to the right of it are included.



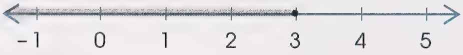
The solution is $x\geq1$.

9. $\frac{2}{3}x\geq x-1$

Y= 2 ÷ 3 X,T,Θ,*n* 2nd [TEST] 4
X,T,Θ,*n* – 1 GRAPH



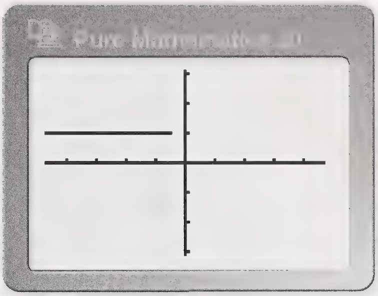
The graph should show that the boundary point and all values to the left of it are included.



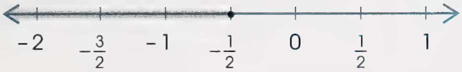
The solution is $x\leq3$.

13. $x - 2 \geq 3x - 1$

Y= X,T,Θ,n − 2 2nd [TEST] 4 3
X,T,Θ,n − 1 GRAPH



The graph should show that the boundary point and all values to the left of it are included.



The solution is $x \leq -\frac{1}{2}$.

7. Textbook questions 1, 2, 3, 4.d., 4.e., 4.f., 5, 6.b., 6.c., 7, 8.b., and 8.c. of Investigation 1, "Inequalities in One Variable," pp. 68 and 69

1. Answers may vary.

   a. Your point should have 3 for the x-coordinate. For example, $(3, 2)$ is a point on the line $x = 3$.

   b. The second point should have the same y-coordinate as your first point and be to the left of $x = 3$. For example, $(2, 2)$ is a point to the left of $(3, 2)$.

   c. The second point has the same y-coordinate as the first point.

   d. The x-coordinate of the second point is less than the x-coordinate of the first point.

2. a. Yes, the points on the line satisfy the inequality because the symbol ≤ means *less than or equal to.*

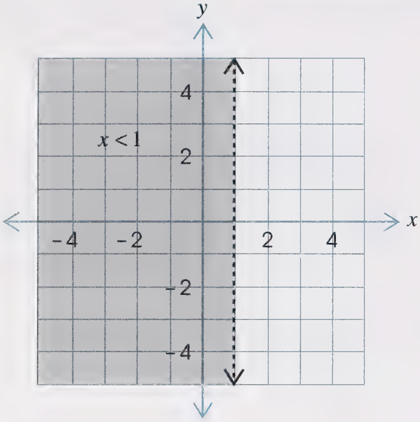
   b. The shaded region consists of every point that has an x-coordinate less than 3. Therefore, the shaded region shows only points that satisfy the inequality.

3. a. Points on the line have x-values equal to 3. Because the symbol < means *less than*, the points on the line do not satisfy the inequality.

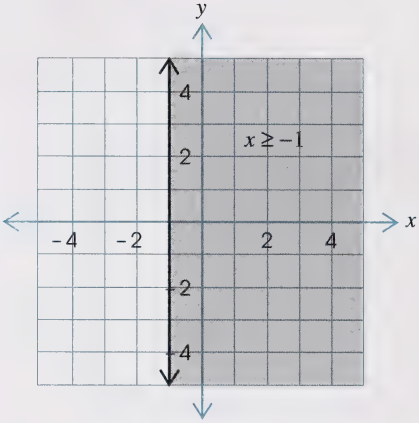
   b. The line $x = 3$ is dotted to indicate it is not included.

Section 2: Activity 1 (continued)

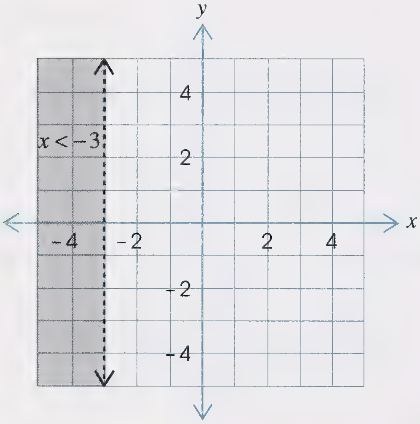
4. d.



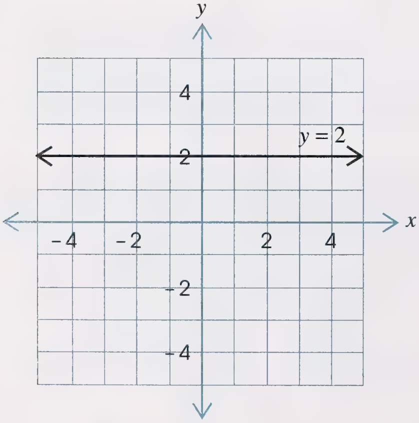
e.



f.

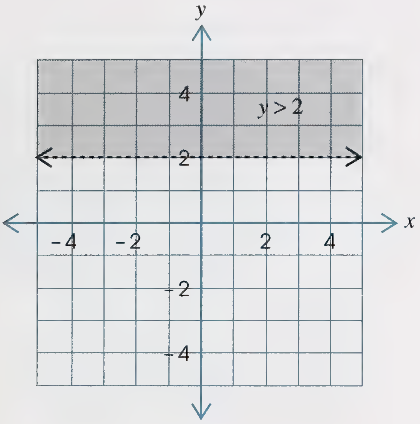


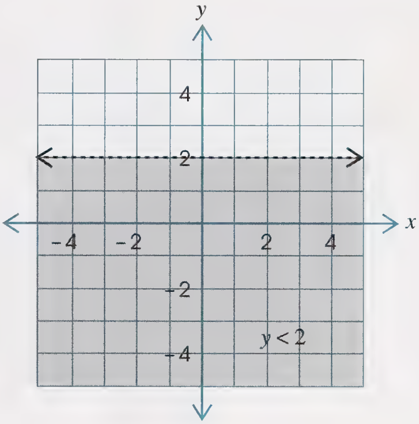
5. a.

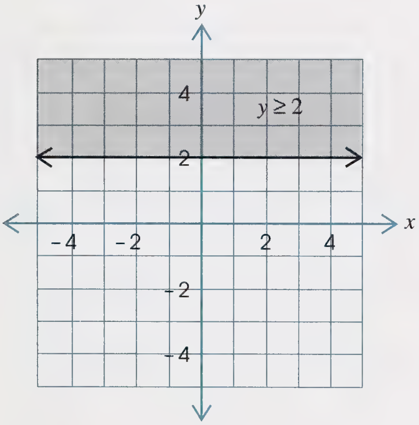


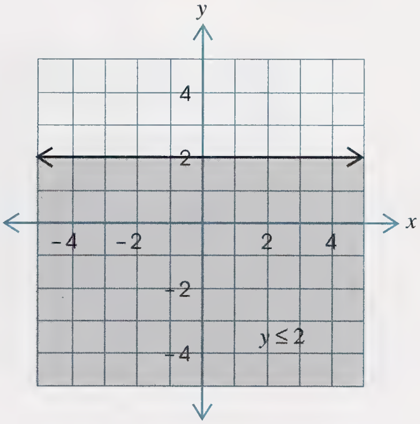
b. The line $y = 2$ is the boundary line for $y > 2$, $y < 2$, $y \geq 2$, and $y \leq 2$.

c.









d. The inequalities $y < 2$ and $y > 2$ do not include the boundary line $y = 2$; therefore, a dotted line is used.

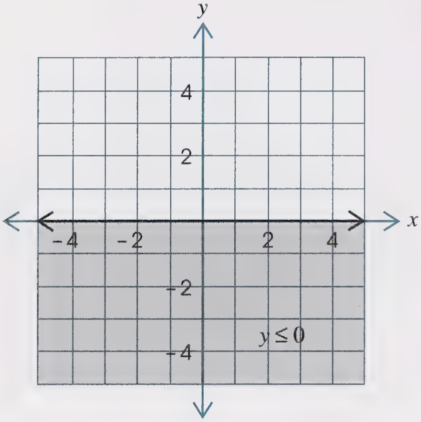
The inequalities $y \leq 2$ and $y \geq 2$ include the boundary line $y = 2$; therefore, a solid line is used.

For $y > 2$ and $y \geq 2$, shade the half-plane above $y = 2$.

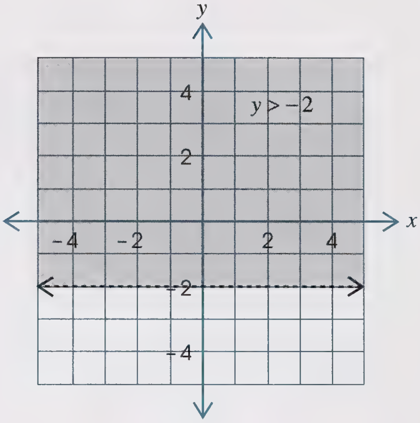
For $y < 2$ and $y \leq 2$, shade the half-plane below $y = 2$.

Section 2: Activity 1 (continued)

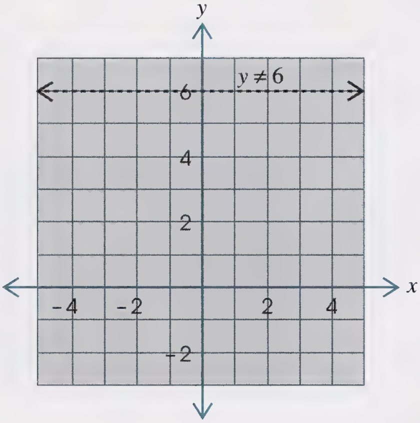
6. b.



c.



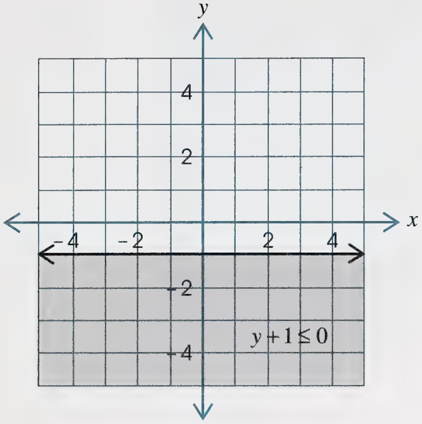
7. a.



b. The inequality $y \neq 6$ is the same as $(y > 6$ or $y < 6)$. That means both planes, above and below the boundary line, must be shaded. Only the boundary line itself is not included. That is why it is shown as a dotted line rather than a solid line.

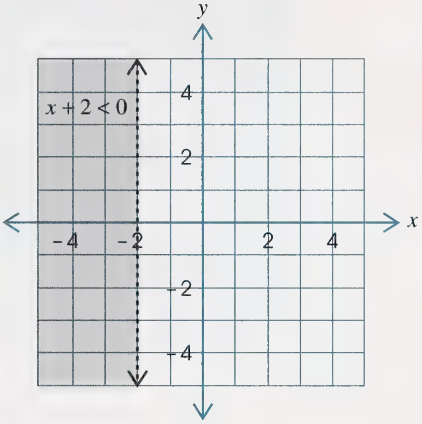
8. **b.** $y+1\leq 0$

$y\leq -1$



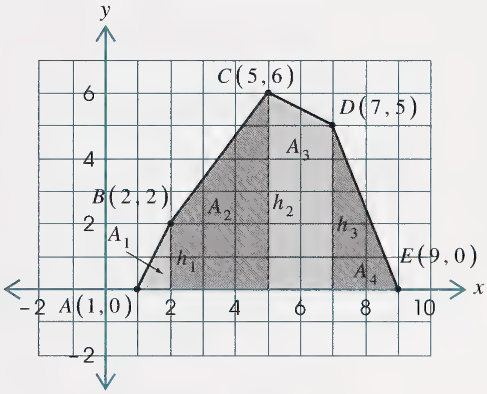
c. $x+2<0$

$x<-2$



8. Textbook questions 1, 2, 3, 6, 9, and 11 of "Applications and Problem Solving," p. 67

1. The area can be found by dividing *ABCDE* into four regions $(A_1, A_2, A_3,$ and $A_4)$ and then finding the sum of the areas of these four regions.



$$A_1=\frac{1}{2}bh_1$$
$$=\frac{1}{2}(1)(2)$$
$$=1$$

$$A_2=b\left(\frac{h_1+h_2}{2}\right)$$
$$=3\left(\frac{2+6}{2}\right)$$
$$=3(4)$$
$$=12$$

$$A_3=b\left(\frac{h_2+h_3}{2}\right)$$
$$=\cancel{2}\left(\frac{6+5}{\cancel{2}}\right)$$
$$=11$$

$$A_4=\frac{1}{2}bh_3$$
$$=\frac{1}{2}(2)(5)$$
$$=5$$

$\therefore$ Area of *ABCDE* $= A_1+A_2+A_3+A_4$

$= 1+12+11+5$

$= 29$ square units

Section 2: Activity 1 (continued)

2.

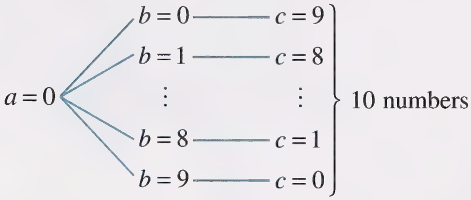
Term Number, *n*	Evaluate	Running Total, *S*(*n*)
1	$1\times2=2$	2
2	$-2\times2=-4$	-2
3	$3\times2=6$	4
4	$-4\times2=-8$	-4
5	$5\times2=10$	6
6	$-6\times2=-12$	-6

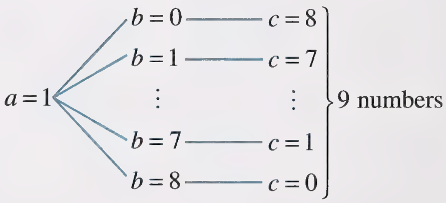
According to the first few rows, $S(n)=-n$ if n is even. Therefore, $S(100)=-100$.

3. This question can be posed as follows:

> How many numbers of the form *abc*, where *a*, *b*, and *c* are whole numbers between 0 and 9, have digits adding up to 9?

Tabulate the possible numbers.





Continuing this pattern,

$a=2$ ⟶ 8 numbers
$a=3$ ⟶ 7 numbers
$a=4$ ⟶ 6 numbers
$a=5$ ⟶ 5 numbers

$a=6$ ⟶ 4 numbers
$a=7$ ⟶ 3 numbers
$a=8$ ⟶ 2 numbers
$a=9$ ⟶ 1 number

$$\begin{aligned}\therefore\ \text{Total} &= 10+9+8+\ldots+3+2+1\\ &= 55\end{aligned}$$

There are 55 numbers that have digits adding up to 9.

6. These are pairs (cards *A* and *B*) that can be formed.

$$\begin{array}{r} \left.\begin{array}{r} 1,\ 2,\ 3,\ldots,44,45,46 \\ 93,92,91,\ldots,50,49,48 \end{array}\right\} 46 \text{ pairs} \\ \hline 94,94,94,\ldots,94,94,94 \phantom{\text{ 46 pairs}} \end{array}$$

You may have over-simplified and obtained 47 by just dividing 94 by 2. You must take into account that 47 has no partner.

9. $n = 2^{23} \times 5^{19}$

$= 2^{4} \times 2^{19} \times 5^{19}$

$= 2^{4} \times 10^{19}$

$= 16 \times 10^{19}$ ← 16 followed by 19 zeros.

Therefore *n* has 21 digits in the expanded form.

11.

$$\begin{array}{rrrrrrr} 1+ & 2+ & 3+\cdots+ & 997+ & 998+ & 999+ & 1000 \\ 999+ & 998+ & 997+\cdots+ & 3+ & 2+ & 1 & \\ \hline 1000+ & 1000+ & 1000+\cdots+ & 1000+ & 1000+ & 1000+ & 1000 \end{array}$$

$\therefore$ Sum $= 1000 \times 1000$

$= 1\,000\,000$

Section 2: Activity 2

1. a. Textbook questions a. and b. of "Explore: Interpret the Data," p. 72

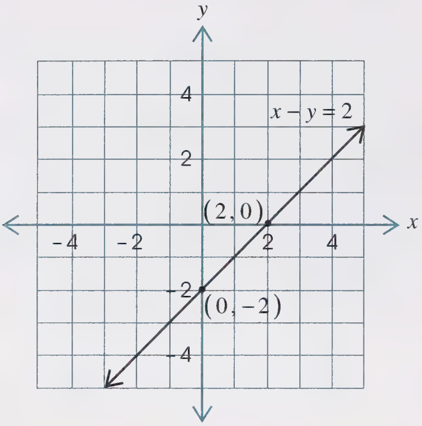
a. If a team either won or lost their games during the regular season, then $x + y = 18$.

b. If a team tied one or more games during the regular season, then $x + y < 18$.

Section 2: Activity 2 (continued)

b. Textbook questions 1 to 6, 7.a., and 7.c. of "Inquire," p. 72

1.



2.

LS	RS
$x - y$	2
$= 0 - 0$	
$= 0$	

LS $\leq$ RS

The point $(0, 0)$ satisfies the inequality.

3. Because the test point $(0, 0)$ lies above the boundary line, the region above the boundary line should be shaded.

4. $x - y \leq 2$

$-y \leq -x + 2$

$y \geq x - 2$

5. Because the inequality indicates greater than or equal to, the region above the boundary line should be shaded.

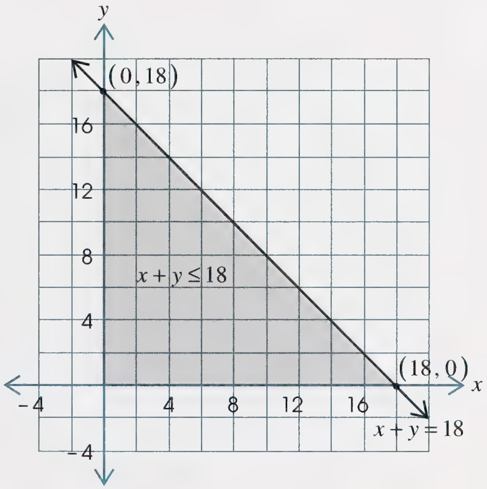
6. Yes, the answers agree.

7. **a.** Test point $(0, 0)$ and see if it satisfies the inequality.

LS	RS
$x+y$	18
$=0+0$	
$=0$	

LS $\leq$ RS

Because point $(0, 0)$ satisfies the inequality, the region below the boundary line is shaded. **Note:** The shading should be restricted to reflect only positive values of x and y.



c. The fact that the test point, $(0, 0)$, satisfies the inequality shows that the shading should be below the boundary line.

2. **Textbook questions 7 to 12 of "Mental Math in Equations and Inequalities," p. 59**

7. $(1,4)$, $(1,10)$, $(1,2)$

8. $(2,-6)$, $(2,-10)$, $(2,-8)$

9. $(0,1)$, $(-1,1)$, $(5,1)$

10. $(-1,3)$, $(0,3)$, $(5,3)$

11. $(1,10)$, $(1,3)$, $(1,5)$

12. $(-2,-100)$, $(-2,0)$, $(-2,7)$

Section 2: Activity 2 (continued)

3. Textbook questions 3, 4, 7, 8, 11, 14, 15, 17, 22, 27, and 28 of “Practice,” p. 75

3. For $(0,0)$,

LS	RS
$2x+3y$	6
$=2(0)+3(0)$	
$=0$	

LS $\ngtr$ RS

For $(3,0)$,

LS	RS
$2x+3y$	6
$=2(3)+3(0)$	
$=6$	

LS $\ngtr$ RS

For $(5,-1)$,

LS	RS
$2x+3y$	6
$=2(5)+3(-1)$	
$=10-3$	
$=7$	

LS $>$ RS

For $(-1,-2)$,

LS	RS
$2x+3y$	6
$=2(-1)+3(-2)$	
$=-2-6$	
$=-8$	

LS $\ngtr$ RS

The ordered pair $(5,-1)$ satisfies the inequality.

4. For $(2,5)$,

LS	RS
$3x-4y$	-12
$=3(2)-4(5)$	
$=6-20$	
$=-14$	

LS $\leq$ RS

For $(-4,0)$,

LS	RS
$3x-4y$	-12
$=3(-4)-4(0)$	
$=-12$	

LS $\leq$ RS

For $(0, 0)$,

LS	RS
$3x - 4y$	-12
$= 3(0) - 4(0)$	
$= 0$	

LS $\nleq$ RS

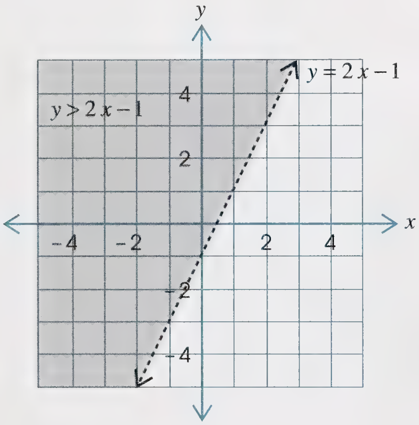
For $(-5, -1)$,

LS	RS
$3x - 4y$	-12
$= 3(-5) - 4(-1)$	
$= -15 + 4$	
$= -11$	

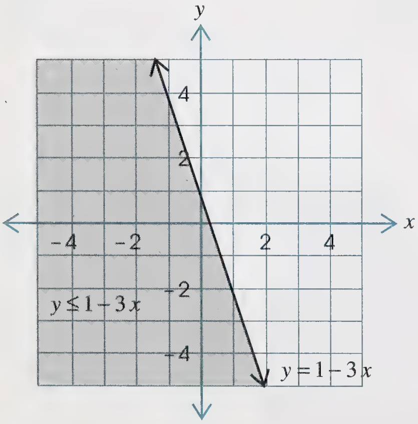
LS $\nleq$ RS

The ordered pairs $(2, 5)$ and $(-4, 0)$ satisfy the inequality.

7.



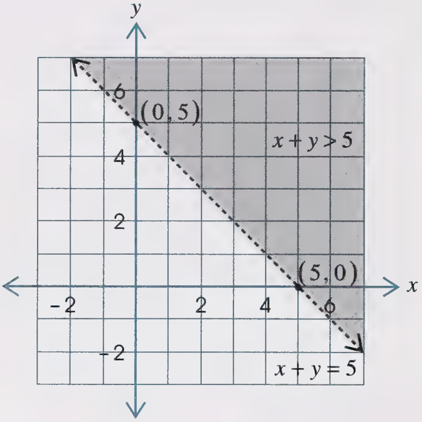
8.



Section 2: Activity 2 (continued)

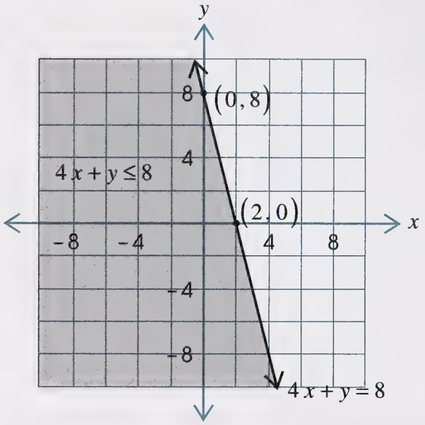
11. $x + y > 5$

$y > 5 - x$



14. $4x + y \leq 8$

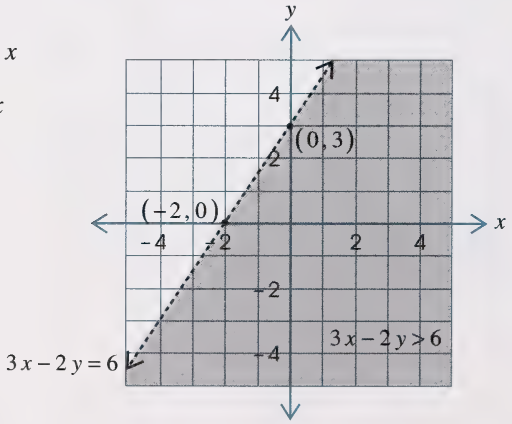
$y \leq 8 - 4x$



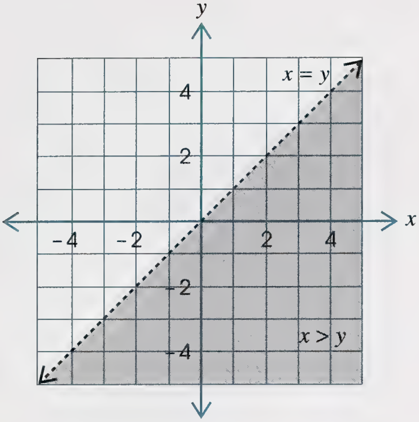
15. $3x - 2y > -6$

$-2y > -6 - 3x$

$y < 3 + \frac{3}{2}x$



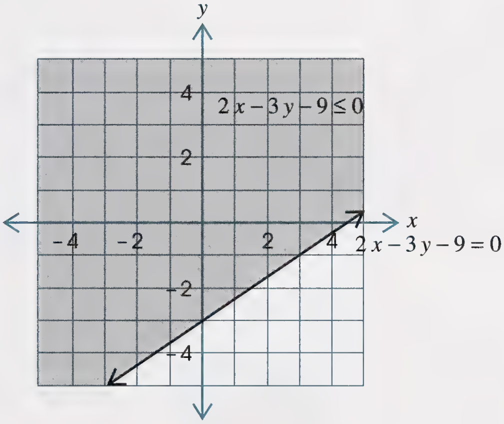
17.



22. $2x - 3y - 9 \le 0$

$$-3y \le -2x + 9$$

$$y \ge \frac{2}{3}x - 3$$



27.

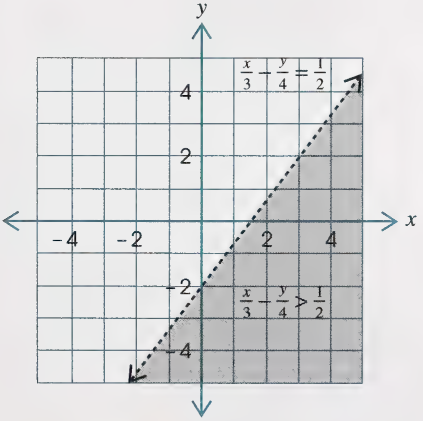
$$\frac{x}{3} - \frac{y}{4} > \frac{1}{2}$$

$$\overset{4}{\cancel{12}}\left(\frac{x}{\underset{1}{\cancel{3}}}\right) - \overset{3}{\cancel{12}}\left(\frac{y}{\underset{1}{\cancel{4}}}\right) > \overset{6}{\cancel{12}}\left(\frac{1}{\underset{1}{\cancel{2}}}\right)$$

$$4x - 3y > 6$$

$$-3y > -4x + 6$$

$$y < \frac{4}{3}x - 2$$



Section 2: Activity 2 (continued)

28.

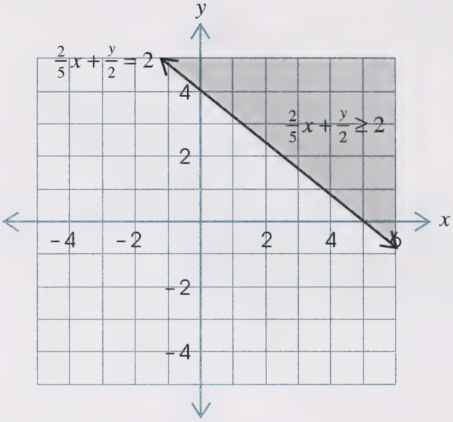
$$\frac{2}{5}x+\frac{y}{2}\ge 2$$

$$\overset{2}{\cancel{10}}\left(\frac{2}{\underset{1}{\cancel{5}}}x\right)+\overset{5}{\cancel{10}}\left(\frac{y}{\underset{1}{\cancel{2}}}\right)\ge 10(2)$$

$$4x+5y\ge 20$$

$$5y\ge 20-4x$$

$$y\ge 4-\frac{4}{5}x$$

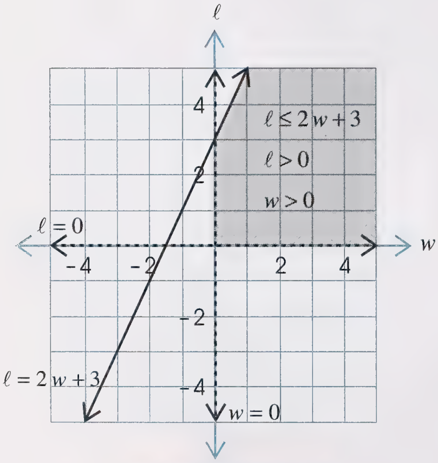


4. **Textbook questions 1, 2, 3, 4.b., 4.d., 4.f., and 7.a. of Investigation 3, "Restricting the Variables," p. 70**

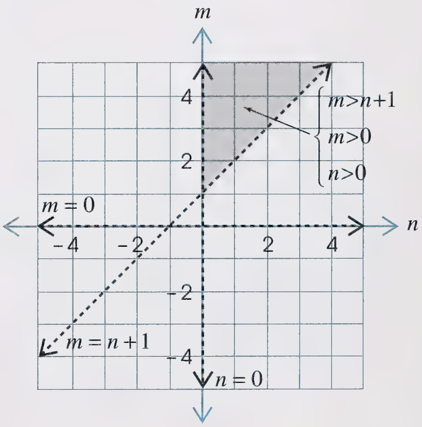
1. **a.** The graph of $b<2a-3$ does not include the *y*-axis. Only part of the half-plane is included—the part to the left of the *y*-axis.

b. The graph of $b<2a-3$ is restricted to negative values of the variables.

2. **a.**



b.



Note: *This graph does not require the length to be greater than the width.*

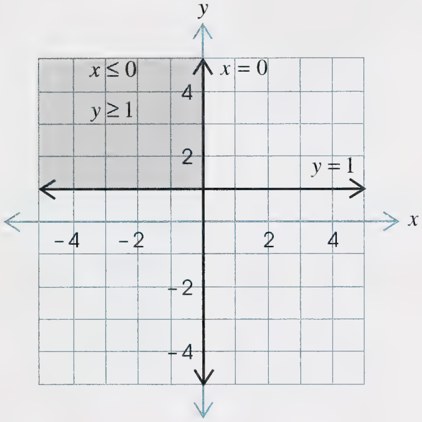
3. **a.** No. The graph requires four conditions, not three.

b. No. The boundary lines on the graph are solid.

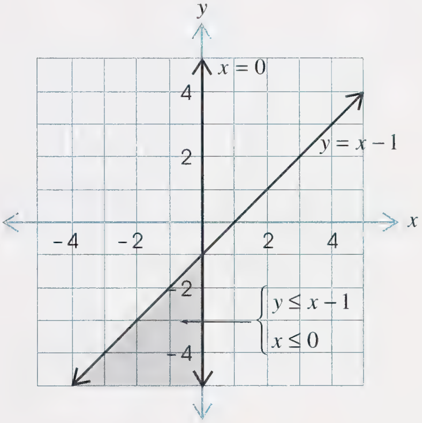
c. Yes. The graph is the region containing points for which $3 \le y \le 5$ and $2 \le x \le 4$.

d. No. The graph requires four conditions, not two.

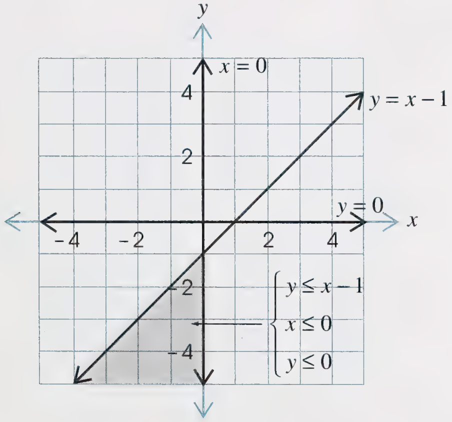
4. **b.**



d.



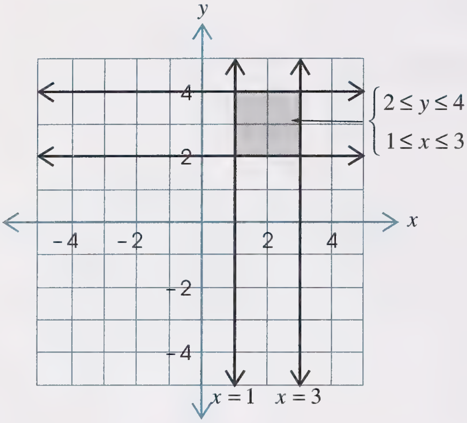
f.



Notice that the graphs in 4.d. and 4.f. are the same. The additional condition, $y \le 0$, in 4.f. did not alter the graph.

Section 2: Activity 2 (continued)

7. a.



From the graph, you can see that the length of the shaded area is 2 units and the width is 2 units.

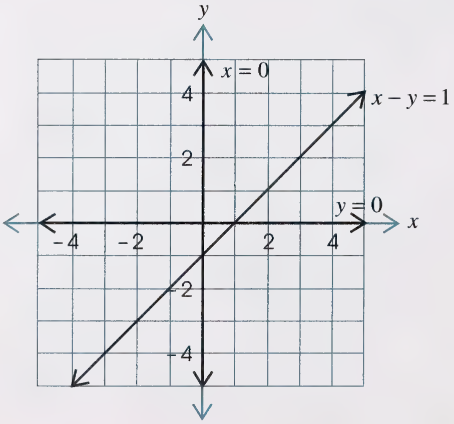
$$\begin{aligned} \therefore \text{ Area} &= \text{length} \times \text{width} \\ &= 2(2) \\ &= 4 \text{ square units} \end{aligned}$$

5. a. Textbook questions 31 and 34 of "Practice," p. 75

31. $x - y \ge 1$

$-y \ge -x + 1$

$y \le x - 1$

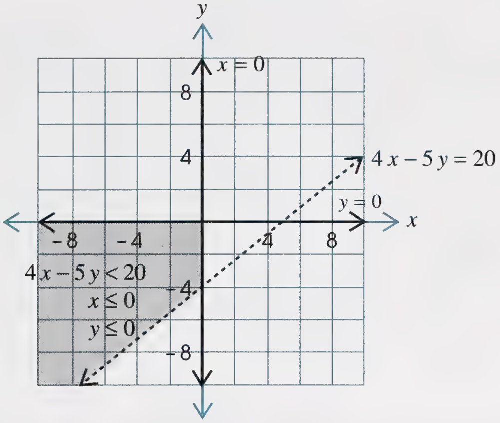


There are no points for which $x - y \ge 1$, $x \le 0$, and $y \ge 0$. Because $x < 0$ and $y > 0$ defines the second quadrant and $x - y > 1$ or $y < x - 1$ lies below the line $x - y = 1$, there is no overlap.

34. $4x - 5y < 20$

$-5y < -4x + 20$

$y > \frac{4}{5}x - 4$

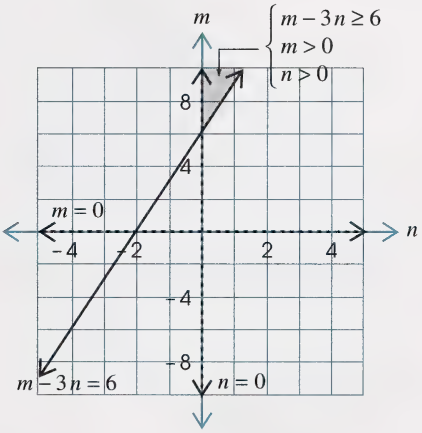


b. Textbook questions 35, 40.a., and 42 of "Applications and Problem Solving," pp. 75 and 76

35. **a.** $m > 0$ and $n > 0$

b. $m - 3n \geq 6$

$m \geq 3n \geq 6$



c. Answers may vary. A sample answer is provided.

$(1, 100)$, $(4, 20)$, and $(3, 15)$

40. **a.** Let x be the number of half-hour appointments, let y be the number of one-hour appointments.

$\therefore\ 0.5x + y \leq 30$, where x and y are whole numbers

Section 2: Activity 2 (continued)

42. **a.** Find the slope of the boundary line. Let $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-1, -4)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4-4}{-1-3} \\ &= \frac{-8}{-4} \\ &= 2 \end{aligned}$$

Use the point-slope formula to find the equation of the line. Let $m = 2$ and $(x_1, y_1) = (3, 4)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= 2(x-3) \\ y - 4 &= 2x - 6 \\ y &= 2x - 2 \end{aligned}$$

Because the boundary line is solid and the shaded half-plane lies below it, the inequality is $y \le 2x - 2$.

b. Let $(x_1, y_1) = (-3, 3)$ and $(x_2, y_2) = (-1, -1)$.

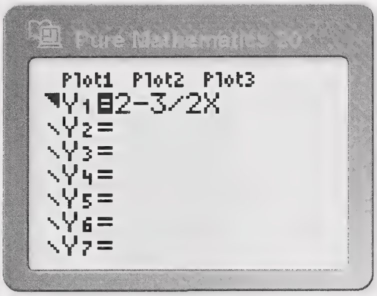
$$\begin{aligned} \therefore\ m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1-3}{-1-(-3)} \\ &= \frac{-4}{2} \\ &= -2 \end{aligned}$$

$$\begin{aligned} \therefore\ y - y_1 &= m(x - x_1) \\ y - 3 &= -2[x - (-3)] \\ y - 3 &= -2(x+3) \\ y - 3 &= -2x - 6 \\ y &= -2x - 3 \end{aligned}$$

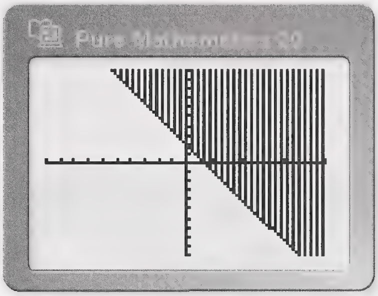
Because the boundary line is dotted and the shaded half-plane lies above it, the inequality is $y > -2x - 3$.

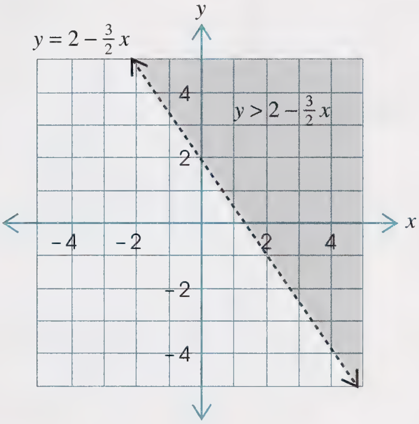
a. **Textbook question 11 of Investigation 1, “Graphing Inequalities,” p. 71**

11.



```
Plot1  Plot2  Plot3
\Y1■2-3/2X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

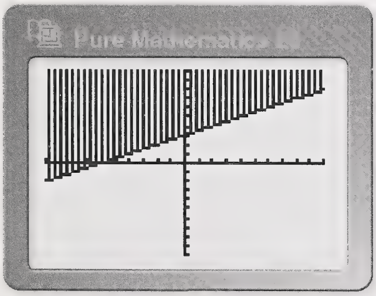


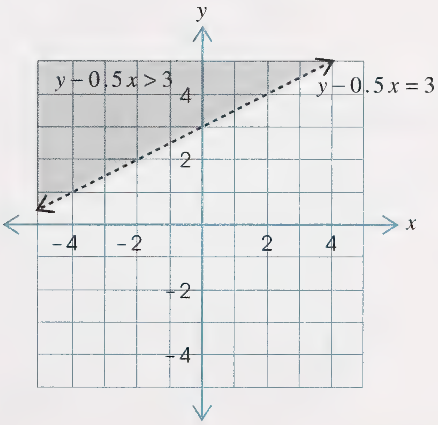


b. **Textbook questions 11 and 13 of Investigation 2, “Solving for *y* and Graphing,” p. 71**

11. $y - 0.5x > 3$

$y > 0.5x + 3$



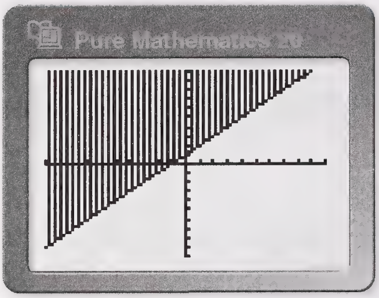


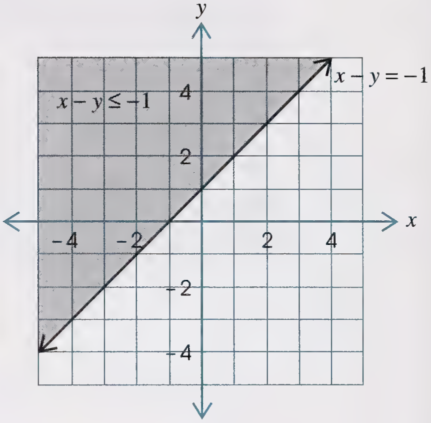
Section 2: Activity 2(continued)

13. $x - y \le -1$

$$-y \le -x - 1$$

$$y \ge x + 1$$





7. **Textbook question 1 of "Applications and Problem Solving," p. 79**

1. 19-L Pail	**13-L Pail**	**7-L Pail**	
0 L	13 L	7 L	
7 L	13 L	0 L	← Empty the 7-L pail into the 19-L pail.
19 L	1 L	0 L	← Fill the 19-L pail from the 13-L pail.
12 L	1 L	7 L	← Fill the 7-L pail from the 19-L pail.
12 L	8 L	0 L	← Empty the 7-L pail into the 13-L pail.

Section 2: Follow-up Activities

Extra Help

1. $-3x \le 6$

$$-3x(-1) \ge 6(-1)$$ ← Multiply both sides by −1, and reverse the inequality.

$$3x \ge -6$$

$$\frac{3x}{3} \ge \frac{-6}{3}$$ ← Divide both sides by 3.

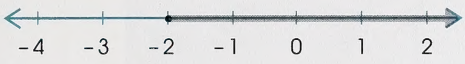
$$x \ge -2$$

Check

Try $x = 3$.

LS	RS
$-3x$	6
$= -3(3)$	
$= -9$	

LS $\leq$ RS



2. $-4x + 4 > 8$

$-4x > 4$ ◄— Subtract 4 from both sides.

$-4x(-1) < 4(-1)$ ◄— Multiply both sides by -1, and reverse the inequality.

$\frac{4x}{4} < \frac{-4}{4}$ ◄— Divide both sides by 4.

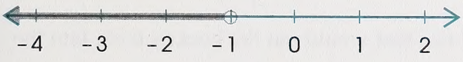
$x < -1$

Check

Try $x = -2$.

LS	RS
$-4x + 4$	8
$= -4(-2) + 4$	
$= 8 + 4$	
$= 12$	

LS $>$ RS

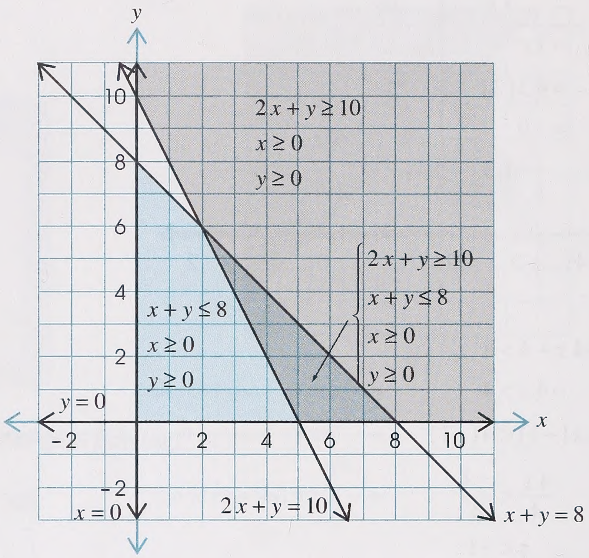


Enrichment

1. a. **Textbook questions a. and b. of "Explore: Draw a Graph," p. 80**

a. $2x + y \geq 10$ ①

$x + y \leq 8$ ②



b. Refer to the answer to question a.

b. **Textbook questions 1, 3, and 4 of "Inquire," p. 80**

1. Because you cannot have a negative number of wins and ties, $x \geq 0$ and $y \geq 0$; therefore, the graph lies only in the first quadrant.

3. a. The number of games the team wins or ties is a whole number. Therefore, the ordered pairs in the solution are made up of whole numbers.

   b. The ordered pairs that satisfy both inequalities are $(2,6)$, $(3,4)$, $(3,5)$, $(4,2)$, $(4,3)$, $(4,4)$, $(5,0)$, $(5,1)$, $(5,2)$, $(5,3)$, $(6,0)$, $(6,1)$, $(6,2)$, $(7,0)$, $(7,1)$, and $(8,0)$.

4. There are 16 different combinations of wins and ties that would put the hockey team into the playoffs.

2. **Textbook questions 7 to 9 of "Practice," p. 85**

7. $y \geq 3 - x$ ①
$y \geq x - 1$ ②

8. $x + y \geq 3$ ①
$x - y \geq 3$ ②

9. $y \geq 3x + 1$ ①
$x + y < 4$ ②

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